The notes written in brackets are just explanations to help you understand the work shown.

2016 FRQs

① (a) [use avg rate of change to approx deriv]

$$R'(2) \approx \frac{R(3)-R(1)}{3-1} = \frac{950-1190}{3-1}$$

$$= -120 \text{ liters}/\text{hr}^2$$

(b) [to get total amt of water removed, we need to integrate the rate at which it is removed.]

$$\int_0^8 R(t)\,dt \approx 1(1340)+2(1190)+3(950)+2(740)$$

$$= 8050 \text{ liters}$$

Since R is a decreasing function a left Riemann sum is an overestimate.

(c) Total water in tank = starting amt + amt added − amt removed

$$50000 + \int_0^8 W(t)\,dt - 8050$$

[given — integrate rate at which water is pumped in to get amt added — from part (b)]

At the end of 8 hours there are approx 49,786 liters in tank.

(d) looking for $W(t) = R(t)$
which means
$W(t) - R(t) = 0$

Since $W(8) - R(8)$ is negative
and
$W(0) - R(0)$ is positive
and we know $W(t) - R(t)$ is continuous then IVT guarantees a value t in $0 < t < 8$ where $W(t) - R(t) = 0$.

(2) (a) [look at signs of $v(t)$ & $a(t)$]

$v(4)$ is $\oplus$
$a(4)$ is $\ominus$

} since $v(4)$ & $a(4)$ have opp signs, the particle is slowing down at $t=4$.

(b) [look at graph of $v(t)$ & find where this graph changes sign]

$v(t)$ changes sign when $t = 2.707$
so the particle changes direction.

(c) $\int_0^4 v(t)\,dt = x(4) - x(0)$

$5.815027 = 2 - x(0)$

$x(0) = -3.815$

(d) $\int_0^3 |v(t)|\,dt = 5.301$

(3) (a) [need to look at g' at $x = 10$]

$$g(x) = \int_2^x f(t)\,dt$$

$$g'(x) = \frac{d}{dx}\int_2^x f(t)\,dt$$

$$g'(x) = f(x)$$

$$g'(10) = f(10) = 0$$

but since there is no sign change in f here, g does not have a rel min or rel max

(b) Since the slope of f changes sign at $x = 4$, g has a point of inflection there

(c) [Use the candidate test. Remember endpt + critical pts are candidates for where abs max + abs mins occur.]

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

[use area under the curve to help get these values of $g(x)$]

[see next page]

$g(-4) = \int_2^{-4} f(t)\,dt$

$= -\int_{-4}^{2} f(t)\,dt$

$= -\left(\frac{1}{2} \cdot 2 \cdot 4\right) = -4$

$g(-2) = \int_2^{-2} f(t)\,dt$

$= -\int_{-2}^{2} f(t)\,dt$

$= -\left(\frac{1}{2} \cdot 4 \cdot 4\right) = -8$

$g(6) = \int_2^{6} f(t)\,dt$

$= \left(\frac{1}{2} \cdot 4 \cdot 4\right) = 8$

$g(12) = \int_2^{12} f(t)\,dt$

$= (8 - 8 + -4) = -4$

The abs min value is -8 & the abs max value is 8 on the interval $-4 \le x \le 12$.

(d) [look for where $g(x) = 0$]

$$g(x) = \int_2^x f(t)\,dt$$

at $x = 2$: $\int_2^2 f(t)\,dt = 0$

at $x = 10$: $\int_2^{10} f(t)\,dt = 0$

[because area of Δ above x axis = area of Δ below x-axis]

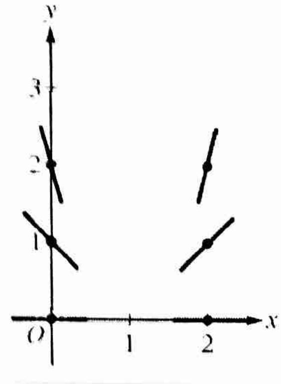
[Now look at part (c) work to see where we will get $g(x)$ to be < 0 in relation to the x values found above.]

$g(x) \le 0$ when $-4 \le x \le 2$

and

$10 \le x \le 12$

(4) (a)



(b) [need slope & pt for tan line]

$$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{3^2}{2-1} = 9 \qquad (2,3) \text{ [was given]}$$

$$y - 3 = 9(x - 2)$$

[use this line to approx y when $x = 2.1$]

$$y - 3 = 9(2.1 - 2)$$

$$y - 3 = 9(0.1)$$

$$y - 3 = 0.9$$

$$y = 3.9 \rightarrow f(2.1) \approx 3.9$$

(c) [separate variables, integrate, find "C", isolate y]

$$\frac{dy}{dx} = \frac{y^2}{x-1}$$

$$\int \frac{dy}{y^2} = \int \frac{dx}{x-1}$$

$$\int y^{-2}\,dy = \int \frac{1}{x-1}\,dx$$

$$\frac{y^{-1}}{-1} = \ln|x-1| + C$$

$$-\frac{1}{y} = \ln|x-1| + C$$

$$-\frac{1}{3} = \ln|2-1| + C$$

$$-\frac{1}{3} = \ln 1 + C$$

$$-\frac{1}{3} = C$$

so $-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$

$$y = \frac{-1}{\ln|x-1| - \frac{1}{3}}$$

(5) (a) [use avg value of a function:
$$\frac{\int_a^b f(x)dx}{b-a}$$
]

$$\frac{\int_0^{10} \frac{1}{20}(3+h^2)\,dh}{10-0} = \frac{1}{200}\cdot\left(3h+\frac{h^3}{3}\right)\Big]_0^{10}$$

$$= \frac{1}{200}\left(3(10)+\frac{100^3}{3}-0\right) \text{ in}$$

or

$$\frac{109}{60} \text{ in}$$

[you do not need to simplify to a single fraction here but you do need units.]

(b) [integrate area of cross section which would be a circle with $r = \frac{1}{20}(3+h^2)$]

$$V = \int_0^{10} \pi\left(\frac{1}{20}(3+h^2)\right)^2 dh$$

$$= \pi \int_0^{10} \frac{1}{400}(9+6h+h^4)\,dh$$

$$= \frac{\pi}{400}\left(9h+2h^2+\frac{h^5}{5}\right)\Big]_0^{10}$$

$$= \frac{\pi}{400}\left(9(10)+2(10)^2+\frac{10^5}{5}-0\right) \text{ in}^3$$

or

$$\frac{2209\pi}{40} \text{ in}^3$$

(c) [related rate question]

$$r = \frac{1}{20}(3+h^2)$$

$$\frac{dr}{dt} = \frac{1}{20}\cdot 2h\frac{dh}{dt}$$

$$-\frac{1}{5} = \frac{1}{20}\cdot 2(3)\frac{dh}{dt}$$

$$-\frac{2}{3} \text{ in/sec} = \frac{dh}{dt}$$

(6) (a) [need slope & pt on line]

$$k(x) = f(g(x))$$
$$k(3) = f(g(3)) = f(6) = 4$$
$$(3, 4)$$

[need chain rule]

$$k'(x) = f'(g(x)) \cdot g'(x)$$
$$k'(3) = f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10$$

$$y - 4 = 10(x-3)$$

(b) [need quotient rule]

$$h'(x) = \frac{g'(x)f(x) - f'(x)g(x)}{(f(x))^2}$$

$$h'(1) = \frac{g'(1)f(1) - f'(1)g(1)}{(f(1))^2}$$

$$= \frac{8 \cdot -6 - 3 \cdot 2}{(-6)^2} \quad \text{or} \quad \frac{-54}{36} \quad \text{or} \quad \frac{-3}{2}$$

(c) $\int_1^3 f''(2x)\,dx$

[need u-sub here with $u = 2x$ so $\frac{du}{2} = dx$]

$$\frac{1}{2} \cdot f'(2x) \Big]_1^3$$
$$= \frac{1}{2}\left(f'(2 \cdot 3) - f'(2 \cdot 1)\right)$$
$$= \frac{1}{2}\left(f'(6) - f'(2)\right)$$
$$= \frac{1}{2}\left(5 - (-2)\right) \quad \text{or} \quad \frac{7}{2}$$