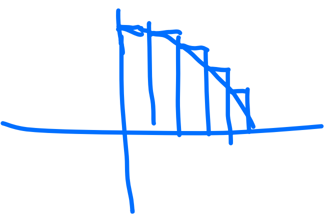
CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2



A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

$2(50.3) + 3(14.4) + 5(6.5) = 176.3 \text{ ft}^3$

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

Overestimate because $A(h)$ is decreasing as h increases.

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

(c) $V = \int_0^{10} f(h)\,dh = 101.325 \text{ ft}^3$

d) $\frac{dh}{dt} = .26$ ft/min

$h = 5$ ft

$V = \int_0^h f(x)\,dx$

$\frac{dV}{dt} = f(h) \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = f(5) \cdot (.26)$

use your calculator

$1.694 \text{ ft}^3/\text{min}$

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$Y_1 \quad f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$Y_2 \quad g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time $t = 8$?

(a) $\int_0^2 f(t)\,dt = 20.051$ lbs

(b) $f'(7) = -8.120$

The rate at which the bananas are being removed is decreasing by 8.120 lb/hr after 7 hours.

(c) $B(t) =$ # of bananas on the display table

$B'(t) = \underbrace{g(t)}_{\text{added}} - \underbrace{f(t)}_{\text{removed}}$

$B'(5) = g(5) - f(5) = -2.263 < 0$

Since $B'(5) < 0$, $B(t)$ is decreasing at $t = 5$ hours

d) $50 + \int_{3}^{8} (g(t))\,dt - \int_{0}^{8} f(t)\,dt = 23.347$

CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

a) $\int_{-6}^{-2} f'(x)\,dx = f(-2) - f(-6)$

$4 = 7 - f(-6)$

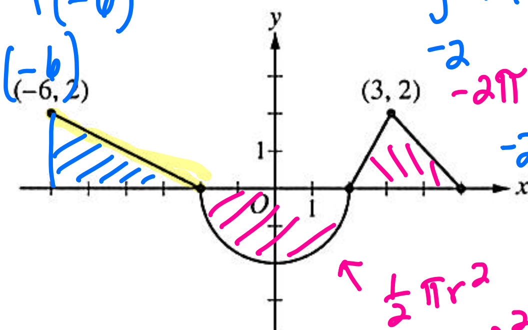
$-3 = -f(-6)$

$3 = f(-6)$

$\int_{-2}^{5} f'(x)\,dx = f(5) - f(-2)$

$-2\pi + 3 = f(5) - 7$

$-2\pi + 10 = f(5)$



$\frac{1}{2}\pi r^2$

$\frac{1}{2}\pi(2)^2$

Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

$(-6,-2)$ and $(2,5)$ b/c $f'(x) > 0$

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

slope of f' at -5 — slope of f' at 3

(c) EVT

$f(-6) = 3$

$f(2) = 7 - 2\pi$

$f(5) = -2\pi + 10$

$\int_{-2}^{2} f'(x)\,dx = f(2) - f(-2)$

$-2\pi = f(2) - 7$

$-2\pi + 7 = f(2)$

abs min: $7 - 2\pi$

d) $f''(-5) = \frac{-2}{4} = -\frac{1}{2}$

$f''(+3) = \text{dne}$ b/c the slope of f' as $x \to 3$ from the right $\neq$ slope of f' as $x \to 3$ from the left