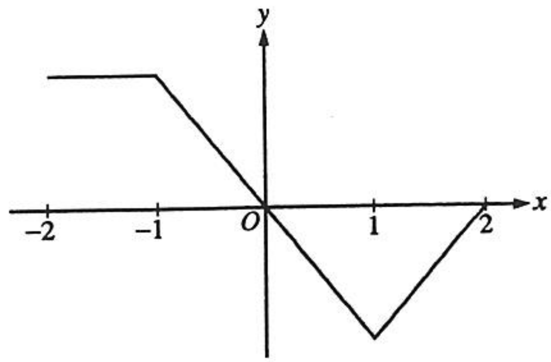
Do Now:
2003

Section I
Part A



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$. maximum

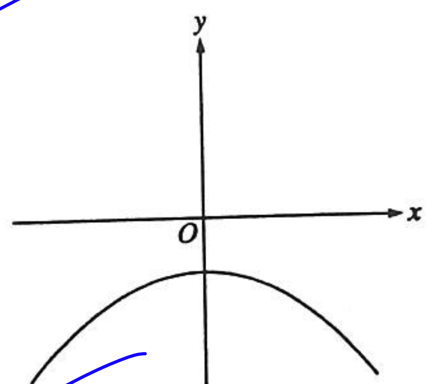
(E) f is not differentiable at $x = -1$ and $x = 1$.

f' is not differentiable at x= ±

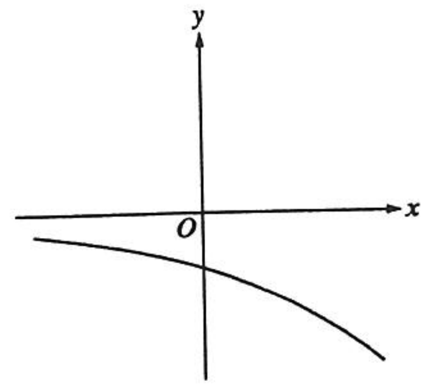
$f(x)$ $\ominus$ graph below x-axis
$f'(x)$ $\ominus$ $f(x)$ $\downarrow$
$f''(x)$ $\ominus$ f concave down

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ?

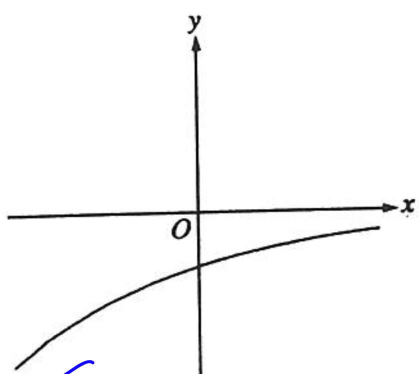
(A)



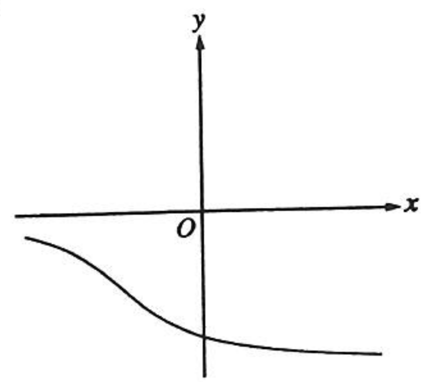
(B)



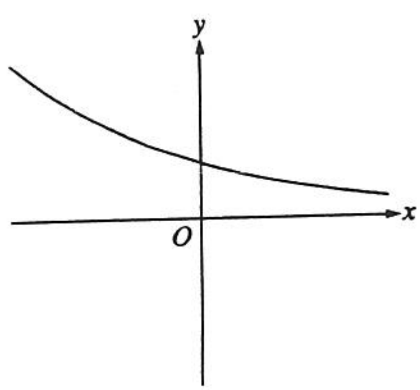
(C)



(D)



(E)



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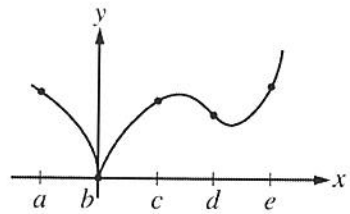

Graph of f

76. The graph of the function f is shown in the figure above. For which of the following values of x is $f'(x)$ positive and increasing?

(A) a (B) b (C) c (D) d (E) e

f' + $f \nearrow$

$f' \nearrow$ f'' + f concave up

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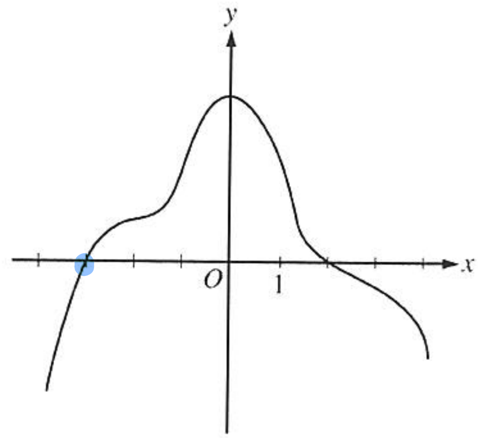

Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$.

II. The graph of f has a point of inflection at $x = -2$.

III. The graph of f is concave down for $0 < x < 4$.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

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Name:______________________________ Date: ______________

AP Calc AB: Testing for relative extrema and points of inflection

Suppose c is a critical point of a continuous function f:

First Derivative Test

1. If f' changes sign from - to + at c, then f has a relative minimum at c.
(local)

2. If f' changes sign from + to - at c, then f has a relative maximum at c.
(local)

For 1-3, find all relative extrema. → rel. mins + max-s

1. $f(x) = x^2 - 3x + 2$

① find critical points (places where $f' = 0$ or dne)

$f'(x) = 2x - 3$

$2x - 3 = 0$

$x = 3/2$

f': − | + (sign change at $\frac{3}{2}$)

f: ↘ $\frac{3}{2}$ ↗

There is a relative minimum of $-\frac{1}{4}$ at $x = 3/2$

$f(3/2) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 2 = \frac{9}{4} - \frac{9}{2} + 2 = \frac{9 - 18 + 8}{4} = \frac{-1}{4}$

2. $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$

$f'(x) = 12x^3 + 12x^2 - 24x$

$12x^3 + 12x^2 - 24x = 0$

$12x(x^2 + x - 2) = 0$

$12x(x+2)(x-1) = 0$

$x = 0, -2, 1$

f': − | + | − | + (sign changes at −2, 0, 1)

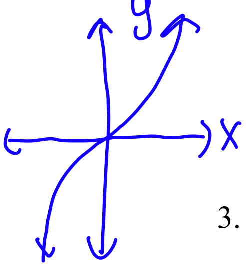
f: ↘ ↗ ↘ ↗

There is a relative min of −30 at $x = -2$

There is a relative max of 2 at $x = 0$

There is a relative min of −3 at $x = 1$

* Remember we are plugging the x values back into f! *

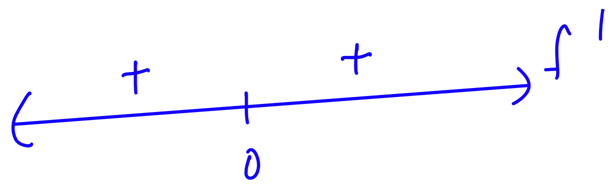


3. $f(x) = x^3$

$f'(x) = 3x^2$

$3x^2 = 0$

$x = 0$



There are no relative extrema

For 4-6, find the open interval on which *f* is concave up and on which *f* is concave down.

4. $f(x) = x^2 - 3x + 2$

$f'(x) = 2x - 3$

$f''(x) = 2$

Since the second derivative is always positive, (here specifically it's 2), the function is always concave up.

CU: $(-\infty, \infty)$

CD: never

there is a pt of inflection at $x = 0$

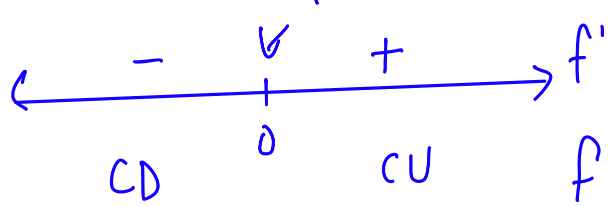
5. $f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$

$6x = 0$

$x = 0$



CD: $(-\infty, 0)$

CU: $(0, \infty)$

6. $f(x) = x^3 - 4x^2 + 1$

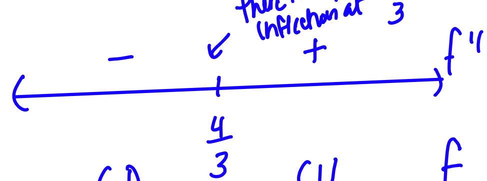
$f'(x) = 3x^2 - 8x$

$f''(x) = 6x - 8$

$6x - 8 = 0$

$x = \frac{4}{3}$

there is a pt of inflection at $\frac{4}{3}$



CD: $(-\infty, \frac{4}{3})$

CU: $(\frac{4}{3}, \infty)$

For 7 and 8 find any inflection points of f.

places where $f'' = 0$ or dne and there is a sign change in f''

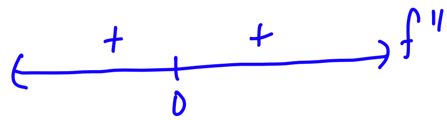
7. $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$12x^2 = 0$

$x = 0$



there are no points of inflection b/c there is no sign change in f''

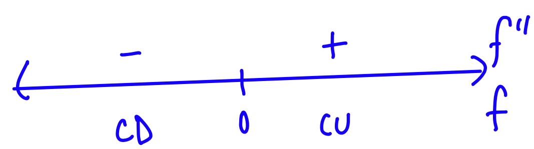
8. $f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$

$6x = 0$

$x = 0$

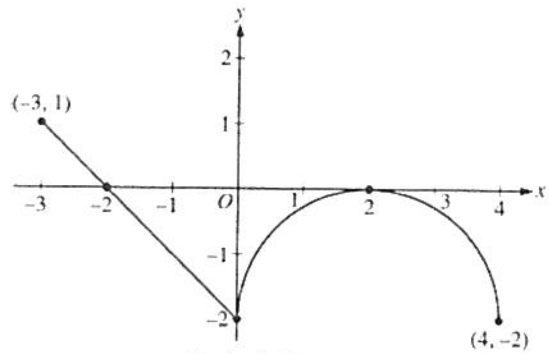


There is a point of inflection at $(0,0)$

Note: If question asks where the relative minimum/maximum occurs, give the x-value. If question asks for the relative minimum/maximum, give the y-value.

Homework 12-05

2003 AB 4



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

(a) f is increasing where f' is positive so $(-3, -2)$

(b) f has a point of inflection where the second derivative changes sign. the second derivative changes sign where f' changes from inc. to dec. or dec. to increasing, so at $x = 0, 2$

(c) $f'(0) = -2$ $\quad$ $y - 3 = -2(x - 0)$