

For each question:

- put the equation in standard form (Show work)
- state the center point
- find and state the vertices and covertices (Show work)
- graph the ellipse
- find and state the foci (Show work)
- state the length of the major axis
- state the length of the minor axis

1. $9x^2 + 25y^2 - 36x - 50y - 164 = 0$

$$9x^2 - 36x + 25y^2 - 50y = 164$$

$$9(x^2 - 4x + 4) + 25(y^2 - 2y + 1) = 164 + 36 + 25$$

$$9(x-2)^2 + 25(y-1)^2 = 225$$

$$\frac{(x-2)^2}{25} + \frac{(y-1)^2}{9} = 1$$

HMA

$$a = 5 \rightarrow$$

$$b = 3 \updownarrow$$

$$c^2 = 25 - 9 = 16$$

$$c = 4 \leftarrow$$

Center: (2, 1)

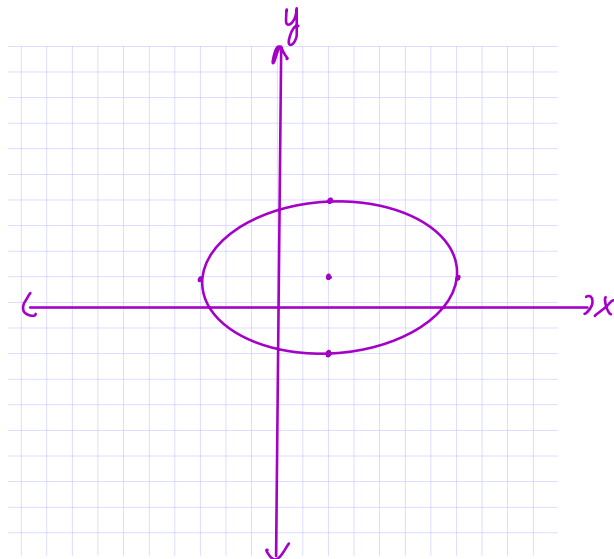
Vertices: $(2 \pm 5, 1)$ $\left\langle \begin{matrix} (7, 1) \\ (-3, 1) \end{matrix} \right.$

Covertices: $(2, 1 \pm 3)$ $\left\langle \begin{matrix} (2, 4) \\ (2, -2) \end{matrix} \right.$

foci: $(2 \pm 4, 1)$ $\left\langle \begin{matrix} (6, 1) \\ (-2, 1) \end{matrix} \right.$

length of major axis: $2(5) = 10$

length of minor axis: $2(3) = 6$



$$\textcircled{2} \quad x^2 + 4y^2 + 6x - 8y + 9 = 0$$

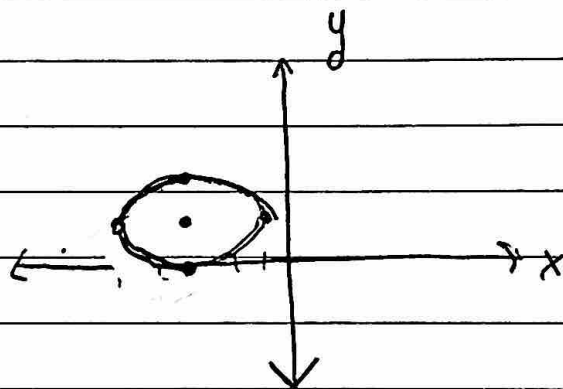
$$\begin{aligned} x^2 + 6x + 9 + 4y^2 - 8y + 4 &= -9 + 9 + 4 \\ x^2 + 6x + 9 + 4(y^2 - 2y + 1) &= -9 + 9 + 4 \\ \frac{(x+3)^2}{4} + \frac{4(y-1)^2}{4} &= \frac{4}{4} \end{aligned}$$

$$\frac{(x+3)^2}{4} + (y-1)^2 = 1$$

Center: $(-3, 1)$ HMA

$$a^2 = 4, \quad a = 2 \Rightarrow$$

$$b^2 = 1, \quad b = 1 \quad \text{TL}$$



$$\text{vertices: } (-3 \pm 2, 1) \begin{cases} (-1, 1) \\ (-5, 1) \end{cases}$$

$$\text{covertices: } (-3, 1 \pm 1) \begin{cases} (-3, 0) \\ (-3, 2) \end{cases}$$

$$c^2 = a^2 - b^2 = 4 - 1 = 3$$

$$c = \sqrt{3} \Rightarrow$$

$$\text{foci: } (-3 \pm \sqrt{3}, 1)$$

$$\text{length of major axis: } 2a = 2(2) = 4$$

$$\text{length of minor axis: } 2b = 2(1) = 2$$

3. $9x^2 + 4y^2 - 36x + 8y + 4 = 0$

$$9x^2 - 36x + 4y^2 + 8y = -4$$

$$9(x^2 - 4x + 4) + 4(y^2 + 2y + 1) = -4 + 9(4) + 4(1)$$

$$\frac{9(x-2)^2}{36} + \frac{4(y+1)^2}{36} = \frac{36}{36}$$

$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$$

center: $(2, -1)$

VMA

$$a^2 = 9, a = 3 \updownarrow$$

$$b^2 = 4, b = 2 \leftarrow$$

$$c^2 = 9 - 4 = 5$$

$$c = \sqrt{5} \updownarrow$$

$$\text{foa: } (2, -1 \pm \sqrt{5})$$

$$\text{length of major axis} = 2(3) = 6$$

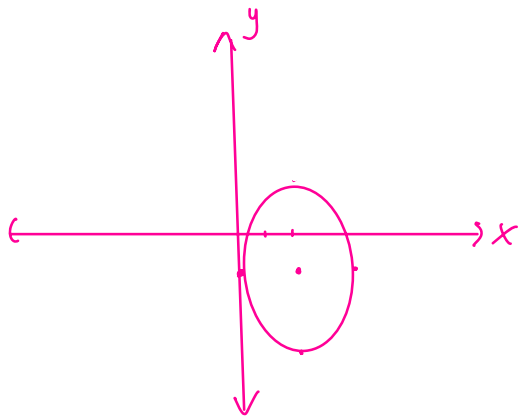
$$\text{length of minor axis} = 2(2) = 4$$

vertices: $(2, -1 \pm 3)$

- $(2, 2)$
- $(2, -4)$

covertices: $(2 \pm 2, -1)$

- $(4, -1)$
- $(0, -1)$



$$\textcircled{4} \quad 4x^2 + y^2 + 4y - 8x - 8 = 0$$

$$\begin{aligned} 4x^2 - 8x + y^2 + 4y + 4 &= 8 \\ 4(x^2 - 2x + 1) + (y + 2)^2 &= 8 + 4 + 4 \\ \frac{4(x-1)^2}{16} + \frac{(y+2)^2}{16} &= \frac{16}{16} \end{aligned}$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

VMA

$$\text{center: } (1, -2)$$

$$a^2 = 16, \quad a = 4 \updownarrow$$

$$b^2 = 4, \quad b = 2 \overleftrightarrow{}$$

$$\text{vertices: } (1, -2 \pm 4) \begin{cases} (1, -6) \\ (1, 2) \end{cases}$$

$$\text{covertices: } (1 \pm 2, -2) \begin{cases} (-1, -2) \\ (3, -2) \end{cases}$$

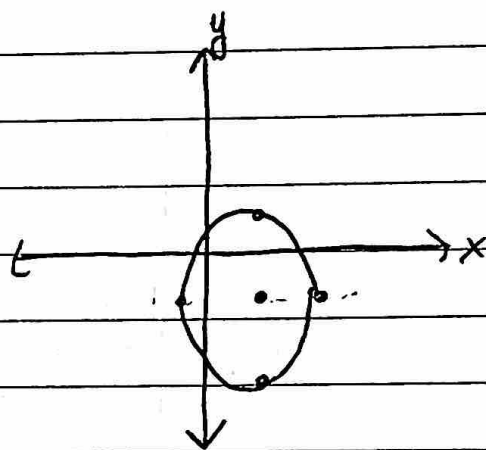
$$c^2 = 16 - 4 = 12$$

$$c = \sqrt{12} \text{ or } 2\sqrt{3} \updownarrow$$

$$\text{foci: } (1, -2 \pm 2\sqrt{3})$$

$$\text{length of major axis: } 2(4) = 8$$

$$\text{length of minor axis: } 2(2) = 4$$



$$5. 9x^2 + 4y^2 + 36x - 24y + 36 = 0$$

$$9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = -36 + 36 + 36$$

$$9(x+2)^2 + 4(y-3)^2 = 36$$

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

center: $(-2, 3)$ vma

$$a = 3 \updownarrow$$

$$b = 2 \rightleftarrows$$

$$c^2 = 5$$

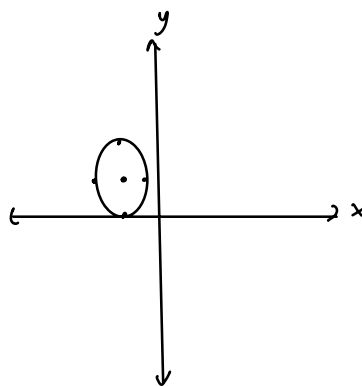
$$c = \pm\sqrt{5} \updownarrow$$

$$\text{vertices: } (-2, 3 \pm 3) \begin{cases} (-2, 6) \\ (-2, 0) \end{cases}$$

$$\text{covertices: } (-2 \pm 2, 3) \begin{cases} (0, 3) \\ (-4, 3) \end{cases}$$

$$\text{foci: } (-2, 3 \pm \sqrt{5})$$

length of major axis: 6
length of minor axis: 4



$$\textcircled{6} \quad 16x^2 - 32x + 25y^2 + 50y = 359$$

$$16(x^2 - 2x + 1) + 25(y^2 + 2y + 1) = 359 + 25 + 16$$

$$16(x-1)^2 + 25(y+1)^2 = 400$$

$$\frac{(x-1)^2}{25} + \frac{(y+1)^2}{16} = 1$$

HMA

$$a = 5$$

$$b = 4$$

$$c^2 = 25 - 16 = 9, \quad c = 3$$

Center $(1, -1)$

$$V: (-4, -1), (6, -1)$$

$$CV: (1, -5), (1, +3)$$

$$F: (1 \pm 3, -1) \quad \begin{matrix} (4, -1) \\ (-2, -1) \end{matrix}$$

major axis length = 10

minor axis length = 8

