Do Now

① Find $\frac{d^2y}{dx^2}$ at $(3,2)$ given $x^2+4y^2=7+3xy$ productrule

$$2x+8y\frac{dy}{dx}=3x\frac{dy}{dx}+3y$$

$$\frac{dy}{dx}(8y-3x)=3y-2x$$

$$\frac{dy}{dx}=\frac{3y-2x}{8y-3x}$$

$$\left.\frac{dy}{dx}\right|_{(3,2)}=\frac{3(2)-2(3)}{8(2)-3(3)}=0$$

$$\left[\frac{3y-2x}{8y-3x}\right]'=\frac{(8y-3x)(3\frac{dy}{dx}-2)-(3y-2x)(8\frac{dy}{dx}-3)}{(8y-3x)^2}$$

$$\frac{(8(2)-3(3))(3(0)-2)-(3(2)-2(3))(8\cdot 0-3)}{(8(2)-3(3))^2}$$

② Given $x^3+y^3=6xy$

a) Find $\frac{dy}{dx}$

$$3x^2+3y^2\frac{dy}{dx}=6y+6x\frac{dy}{dx}$$

$$\frac{dy}{dx}(3y^2-6x)=6y-3x^2$$

$$dy/dx=\frac{6y-3x^2}{3y^2-6x}=\frac{2y-x^2}{y^2-2x}$$

b) Find the equation of the tangent line to the curve at the point $(3,3)$.

$$\left.\frac{dy}{dx}\right|_{(3,3)}=\frac{2(3)-3^2}{3^2-2(3)}=\frac{-3}{3}=-1$$

$$y-3=-1(x-3)$$

c) At what point(s) on the curve is/are the tangent line(s) horizontal? $\frac{dy}{dx}=0$

$$\frac{2y-x^2}{y^2-2x}=0$$

$$2y-x^2=0$$

$$y=\frac{x^2}{2}$$

$$x^3+\left(\frac{x^2}{2}\right)^3=\overset{3}{\cancel{6}}x\left(\frac{x^2}{\cancel{2}}\right)$$

$$8\left(x^3+\frac{x^6}{8}=3x^3\right)$$

$$8x^3+x^6=24x^3$$

$$x^6-16x^3=0$$

$$x^3(x^3-16)=0$$

$x^3=0, \quad x=0$

$x^3-16=0$

$x^3=16$

$x=\sqrt[3]{16}$

$(0,0)$

$\left(\sqrt[3]{16}, \frac{(\sqrt[3]{16})^2}{2}\right)$

d) At what point(s) on the curve is/are the tangent line(s) vertical? $\frac{dy}{dx}$ to be undefined (set denominator $=0$)

$$y^2-2x=0$$

$$x=\frac{y^2}{2}$$

$$\left(\frac{y^2}{2}\right)^3+y^3=\overset{3}{\cancel{6}}\left(\frac{y^2}{\cancel{2}}\right)y$$

$$\frac{y^6}{8}+y^3=3y^3$$

$$y^6+8y^3=24y^3$$

$$y^6-16y^3=0$$

$$y^3(y^3-16)=0$$

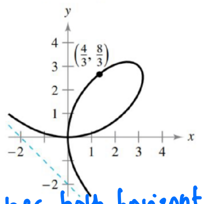
$\cancel{y=0}$ | $y=\sqrt[3]{16}$

$(0,0)$

$\left(\frac{(\sqrt[3]{16})^2}{2}, \sqrt[3]{16}\right)$

Folium of Descartes

$x^3+y^3-6xy=0$



has both horizontal + vertical tan lines at (0,0)

③ Differentiate:

a) $y=\frac{6^{\pi x}}{2x^2-3}$

$$y'=\frac{(2x^2-3)\cdot 6^{\pi x}\cdot \ln 6\cdot \pi-6^{\pi x}(4x)}{(2x^2-3)^2}$$

b) $\log_{10}\left(\frac{x}{x-1}\right)$

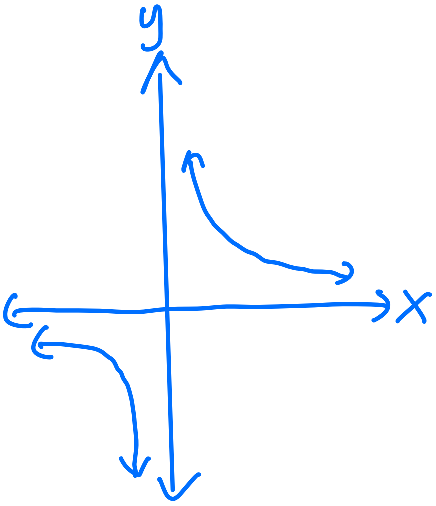
$\overset{x>0}{\log_{10}x}-\overset{x>1}{\log_{10}(x-1)}$ $x>1$

$$\frac{1}{x\ln 10}\cdot 1-\frac{1}{(x-1)\ln 10}\cdot 1$$

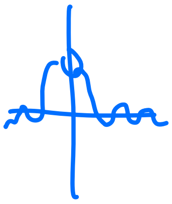
$$\frac{1}{x\ln 10}-\frac{1}{(x-1)\ln 10}$$

Exam 1

㉑ $$\lim_{x\to 0}\left(x^3\sin\left(\frac{1}{x}\right)\right)$$



$u=\frac{1}{x}$, $x=\frac{1}{u}$

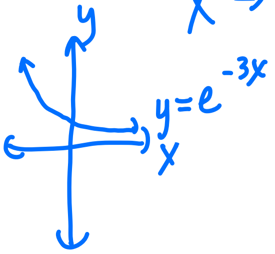


$$\lim_{u\to\pm\infty}\frac{1}{u^3}\sin u$$

$$\lim_{u\to\pm\infty}\left(\frac{\sin u}{u}\cdot\frac{1}{u^2}\right)=0$$

$$\lim_{u\to\infty}\left(\overset{0}{\frac{\sin u}{u}}\cdot\overset{0}{\frac{1}{u^2}}\right)=0 \qquad \lim_{u\to-\infty}\left(\overset{0\,\cdot}{\frac{\sin u}{u}}\cdot\overset{0}{\frac{1}{u^2}}\right)=0$$

⑭ $$\lim_{x\to\infty}\left(e^{-3x}\sin x\right)$$



$$\underset{0}{\lim_{x\to\infty}e^{-3x}}\cdot\underset{\substack{\text{dne}\\ \uparrow\\ \text{bounded}}}{\lim_{x\to\infty}\sin x}=0$$

Exam 3

⑤ $y' = 0$

$$y = 2xe^{-x}$$

$$y' = 2x \cdot e^{-x}(-1) + 2e^{-x}$$

$$y' = -2xe^{-x} + 2e^{-x}$$

$$y' = 2e^{-x}(-x+1)$$

$$2e^{-x}(-x+1) = 0$$

$2e^{-x} \neq 0$	$-x+1 = 0$ $1 = x$

Exam 1

⑲ $$\lim_{x \to 3^-} \frac{\overset{.1}{3-x}}{\underset{.1}{|x-3|}} = 1$$

$$\begin{cases} \dfrac{3-x}{x-3} = -1 & x-3>0,\ x>3 \\ \dfrac{3-x}{-(x-3)} = 1 & x<3 \end{cases}$$

Exam 3

⑦ $$f(x) = (x-1)^{3/2} + \frac{e^{x-2}}{2} \rightarrow \frac{1}{2}e^{x-2} \qquad f'(2)$$

$$f'(x) = \frac{3}{2}(x-1)^{\frac{1}{2}} + \frac{1}{2}e^{x-2}$$

$$f'(2) = \frac{3}{2}(2-1)^{\frac{1}{2}} + \frac{1}{2}e^{2-2}$$

$$= \frac{3}{2} + \frac{1}{2} = \frac{4}{2}$$

Exam 2 14

$(1,5)$

#14 $g(x) = x^3 + 3x^2 - x + 2$

$g'(x) = 3x^2 + 6x - 1$

$g'(1) = 3(1)^2 + 6(1) - 1 = 8$

$y - 5 = 8(x-1)$

$y = 8x - 8 + 5 = 8x - 3$

$8x - 3 = x^3 + 3x^2 - x + 2$

$0 = x^3 + 3x^2 - 9x + 5$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -9 & 5 \\ & & 1 & 4 & -5 \\ \hline & 1 & 4 & -5 & 0 \end{array}$$

$(x-1)(x^2 + 4x - 5) = 0$

$(x-1)(x+5)(x-1) = 0$

$x = 1 \quad | \quad x = -5 \quad | \quad x = 1$

Q

$(-5, -43)$

$\uparrow$

$8(-5) - 3$

Secret Number: $-4+(-1)+\cancel{2}+(\cancel{-2})+(-5)+0+\cancel{2+(-2)}=-10$

) $x^3+xy-2=0$

$3x^2+x\,dy/dx+y=0$ $(1,1)$

$3(1)^2+1\,dy/dx+1=0$

$3+dy/dx+1=0$

$4+dy/dx=0$

$\boxed{dy/dx=-4}$

② $x^3+y^3=6xy$ $(3,3)$

$3x^2+3y^2\,dy/dx=6x\,dy/dx+6y$

$3(3)^2+3(3)^2\,dy/dx=6(3)\,dy/dx+6(3)$

$27+27\,dy/dx=18\,dy/dx+18$

$9\,dy/dx=-9$

$\boxed{dy/dx=-1}$

) $x^3+y^3=8$

$3x^2+3y^2\,dy/dx=0$

$3y^2\,dy/dx=-3x^2$

$dy/dx=-x^2/y^2$

$dy/dx\Big|_{(0,2)}=0$

$\boxed{y=2}$

④ $x=y^2$ @ $y=1/2$, $x=1/4$

$1=2y\,dy/dx$

$\frac{1}{2y}=dy/dx$

$\left[\frac{1}{2y}\right]'=\frac{-2\,dy/dx}{(2y)^2}=\frac{-\cancel{2}\,dy/dx}{\underset{2}{\cancel{4}}y^2}$

$\frac{-\frac{1}{2(1/2)}}{2\left(\frac{1}{2}\right)^2}=\frac{-1}{1/2}$

$\boxed{=-2}$

) $x^2-xy+y^2=7$ $(1,3),(1,-2)$

$1-y+y^2=7$

$y^2-y-6=0$

$(y-3)(y+2)=0$

$y=3 \mid y=-2$

$2x-y-x\,dy/dx+2y\,dy/dx=0$

$dy/dx(2y-x)=y-2x$

$dy/dx=\frac{y-2x}{2y-x}$

$\frac{dy}{dx}\Big|_{(1,3)}=\frac{3-2(1)}{2(3)-1}=\frac{1}{5}$ $\boxed{m_n=-5}$

$\frac{dy}{dx}\Big|_{(1,-2)}=\frac{-2-2(1)}{2(-2)-1}=\frac{-4}{-5}$ $m_n=\frac{5}{4}$

⑥ $x^3 - 6xy - ky^3 = a$

$3x^2 - 6x\,dy/dx - 6y - 3ky^2\,dy/dx = 0$

$dy/dx(-6x - 3ky^2) = -3x^2 + 6y$

$$\frac{dy}{dx} = \frac{-3x^2 + 6y}{-6x - 3ky^2}$$

$\frac{3x^2 + 6y}{-6x - 3ky^2} = -1$ @ $(0,1)$ $\Rightarrow$ $\frac{3(0)^2 + 6(1)}{-6(0) - 3k(1)^2} = -1$

$\frac{6}{-3k} = -1$

$3k = 6$

$k = 2$

$x^3 - 6xy - ky^3 = a$

$x^3 - 6xy - 2y^3 = a$ @ $(0,1)$

$0 - 6(0)(1) - 2(1)^3 = a$

$-2 = a$

$\boxed{-2 + 2 = 0}$

⑦ $x^2 + 4y^2 = 7 + 3xy$

$2x + 8y\,dy/dx = 3x\,dy/dx + 3y$

$dy/dx(8y - 3x) = 3y - 2x$

$$\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$$

$3y - 2x = 0$ when $x = 3$

$3y - 2(3) = 0$

$3y - 6 = 0$

$3y = 6$

$\boxed{y = 2}$

⑧ $x^2 + 2x + y^4 + 4y = 5$

$2x + 2 + 4y^3\,dy/dx + 4\,dy/dx = 0$

$dy/dx(4y^3 + 4) = -2x - 2$

$$\frac{dy}{dx} = \frac{-2x - 2}{4y^3 + 4} = \frac{-x - 1}{2y^3 + 2}$$

$2y^3 + 2 = 0$

$2y^3 = -2$

$y^3 = -1$

$y = -1$

$x^2 + 2x + (-1)^4 + 4(-1) = 5$

$x^2 + 2x + 1 - 4 = 5$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$ $x = -4, 2$

$\boxed{-4 + 2 = -2}$