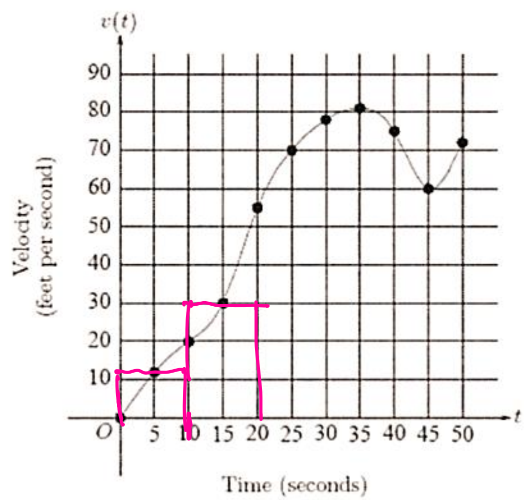
Name:________________________ Date:________

AP Calculus AB Ms. Loughran

1998 AP Calculus AB Free-Response Questions



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

(a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer. (0,35), (45,50) b/c v(t) is inc. over those intervals

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

(d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

b) $\frac{v(50)-v(0)}{50} = \frac{72-0}{50} = \frac{72}{50}$ ft/sec^2

c) $a(40) \approx \frac{v(45)-v(35)}{45-35} = \frac{60-81}{10} = -\frac{21}{10}$ ft/sec^2

d) $\int_0^{50} v(t)dt = 10[12+30+70+81+60]$ ft

the total distance traveled in ft in the intervals 0 to 50 seconds

2003 AB 4

$\int_0^4 f'(x)\,dx = f(4) - f(0)$

$= f(4) - 3$

$-(A_\square - A_\triangledown) = f(4) - 3$

$-\left(8 - \frac{1}{2}\pi(2)^2\right) = f(4) - 3$

$-(8 - 2\pi) = f(4) - 3$

$-8 + 2\pi = f(4) - 3$

$-5 + 2\pi = f(4)$

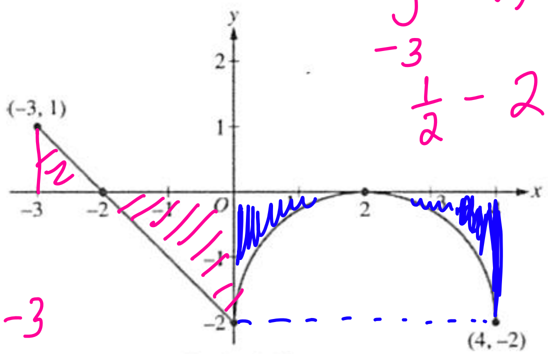
d) $\int_{-3}^{0} f'(x)\,dx = f(0) - f(-3)$

$= 3 - f(-3)$

$\frac{1}{2} - 2$

$-\frac{3}{2} = 3 - f(-3)$

$f(-3) = \frac{9}{2}$



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

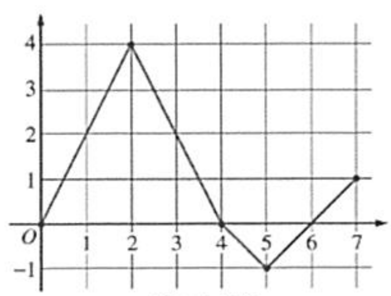
(a) On what intervals, if any, is f increasing? Justify your answer. (−3, −2) b/c f' is positive in that intervals.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer. f'' + − or − +, f' ↗ ↘ or ↘ ↗ $x = 0, 2$

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$. $f'(0) = -2$ $y - 3 = -2x$

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

2003 AB 5 Form B



Graph of f

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

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AP Calculus: The Definite Integral as an Average Ms. Loughran

We know how to find the average of n numbers: add them and divide by n. But how do we find the average value of a continuously varying function? Let us consider an example. Suppose $C = f(t)$ is the temperature at time t, measured in hours since midnight, and that we want to calculate the average temperature over a 24-hour period. One way to start would be to average the temperatures at n times, $t_1, t_2, \ldots, t_n$, during the day.

$$\text{Average temperature} \approx \frac{f(t_1) + f(t_2) + \cdots + f(t_n)}{n}.$$

The larger we make n, the better the approximation. We can rewrite this expression as a Riemann sum over the interval $0 \le t \le 24$ if we use the fact that $\Delta t = 24/n$, so $n = 24/\Delta t$:

$$\begin{aligned}\text{Average temperature} &\approx \frac{f(t_1) + f(t_2) + \cdots + f(t_n)}{24/\Delta t} \\ &= \frac{f(t_1)\Delta t + f(t_2)\Delta t + \cdots + f(t_n)\Delta t}{24} \\ &= \frac{1}{24}\sum_{i=1}^{n} f(t_i)\Delta t.\end{aligned}$$

As $n \to \infty$, the Riemann sum tends towards an integral and also approximates the average temperature better. Thus, in the limit

$$\begin{aligned}\text{Average temperature} &= \lim_{n\to\infty} \frac{1}{24}\sum_{i=1}^{n} f(t_i)\Delta t \\ &= \frac{1}{24}\int_0^{24} f(t)\,dt.\end{aligned}$$

Thus we have found a way of expressing the average temperature in terms of an integral. Generalizing for any function f, we define

[a,b]

$$\boxed{\text{Average value of } f \text{ from } a \text{ to } b = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

average velocity $= \dfrac{x(b) - x(a)}{b-a} = \dfrac{1}{b-a}\displaystyle\int_a^b v(t)dt$

average acceleration : $\dfrac{v(b)-v(a)}{b-a} = \dfrac{1}{b-a}\displaystyle\int_a^b a(t)dt$

How to Visualize the Average on a Graph

The definition of average value tells us that

$$(\text{Average value of } f)\cdot(b-a) = \int_a^b f(x)\,dx.$$

Thus, if we interpret the integral as the area under the graph of f, then we can think of the average value of f as the height of the rectangle with the same area that is on the same base, $(b-a)$. (See Figure 3.18.)

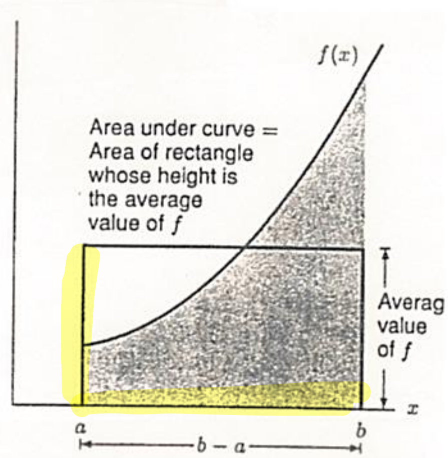


Figure 3.18: Area and average value

Practice

1. Find the average value of $f(x) = x^2$ from $x = 2$ to $x = 4$.

$$\frac{1}{4-2}\int_2^4 x^2\,dx = \frac{1}{2}\left[\frac{x^3}{3}\Big|_2^4\right]$$

$$\frac{1}{2}\left[\frac{4^3}{3} - \frac{2^3}{3}\right] = \frac{1}{2}\left[\frac{64}{3} - \frac{8}{3}\right]$$

$$\frac{1}{\cancel{2}}\left[\frac{\cancel{56}}{3}\right] = \frac{28}{3}$$

2. Find the average value of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 16$.

$$\frac{1}{16-0}\int_0^{16} \sqrt{x}\,dx = \frac{1}{16}\left[\frac{2}{3}x^{3/2}\Big|_0^{16}\right] = \frac{1}{16}\left[\frac{2}{3}(64) - 0\right]$$

$$\frac{1}{\cancel{16}}\cdot\frac{2}{3}(\cancel{64}^{4}) = \frac{8}{3}$$

3. Find the average value of $f(x) = e^{2x}$ on the interval [-1,1].

$$\frac{1}{1-(-1)}\int_{-1}^{1} e^{2x}\,dx = \frac{1}{2}\left[\frac{1}{2}e^{2x}\Big|_{-1}^{1}\right] = \frac{1}{4}\left[e^{2} - e^{-2}\right]$$

$$\frac{e^2}{4} - \frac{1}{4e^2}$$

4. Find the average value of $f(x) = \cos x$ from $x = 0$ to $x = \pi$.

$$\frac{1}{\pi}\int_0^{\pi} \cos x\,dx = \frac{1}{\pi}\sin x\Big|_0^{\pi}$$

$$\frac{1}{\pi}(\sin\pi - \sin 0) = 0$$

5. Find the average velocity of $v(t) = t^2 - 2$ on the interval [-2,3].

$$\frac{1}{3-(-2)}\int_{-2}^{3} (t^2-2)\,dt = \frac{1}{5}\left[\frac{t^3}{3} - 2t\Big|_{-2}^{3}\right]$$

$$\frac{1}{5}\left[9 - 6 - \left(\frac{-8}{3} + 4\right)\right]$$

$$\frac{1}{5}\left[3 + \frac{8}{3} - 4\right] = \frac{1}{5}\left[\frac{5}{3}\right] = \frac{1}{3}$$

Homework 03-07

2002 AB 4

a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\frac{(1)(3)}{2} = -\frac{3}{2}$

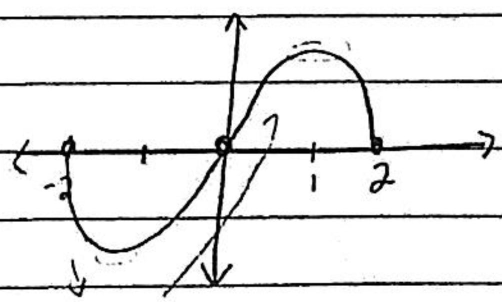
$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = \frac{3-(-3)}{0-(-2)} = \frac{6}{2} = 3$

(b) g increases when $g'(x) > 0$ so $g'(x) = f(x) > 0$ from $(-1, 1)$

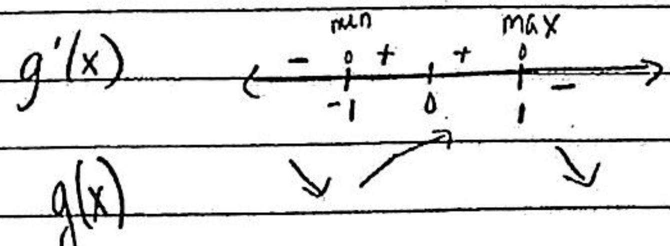
(c) g is concave down when $g''(x) < 0$, $g'(x) = f(x)$ is decreasing so from $(0, 2)$.

(d)



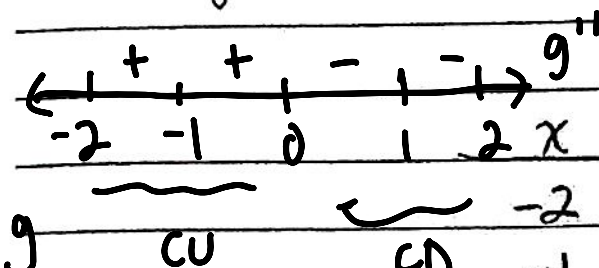
$g(2) = \int_0^2 f(t)\,dt$

$\frac{3 \cdot 1}{2} - \frac{3 \cdot 1}{2} = 0$



$g(-2) = 0$

$g(0) = \int_0^0 f(t)\,dt = 0$



x	g(x)
-2	0
-1	-3/2
0	0
1	3/2

make a table of values

2 0

(2002 AB4 Form B)

(a) $g(6) = 5 + \int_6^6 f(t)\,dt = 5 + 0 = 5$

$g'(6) = f(6) = 3$

$g''(6) = \frac{1-1}{3-9} = 0$ derivative of g' horizontal tan line

(b) g is decreasing when $g'(x) = f(x) < 0$ so $[-3, 0]$ and $[12, 15]$

(c) g is concave down when $g''(x) < 0$, $g'(x) = f(x)$ is decreasing, so $(6, 15)$

(d) $\int_{-3}^{15} f(t)\,dt = \frac{(0+(-1))3}{2} + \frac{(1+0)3}{2} + \frac{(3+1)3}{2} + \frac{(1+3)3}{2} + \frac{(0+1)3}{2} + \frac{(-1+0)3}{2}$

$-\cancel{3/2} + \cancel{3/2} + 6 + 6 + \cancel{3/2} - \cancel{3/2}$

12

x	-3	0	3	6	9	12	15
$f(x)$	-1	0	1	3	1	0	-1