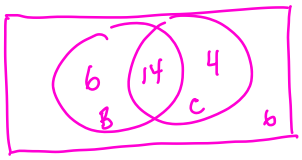
Do Now:
#15 from the Venn Diagram Practice sheet

15 There are 30 students on a school bus. Of these students, 24 either play in the school band or sing in the chorus. Six of the students play in the school band but do not sing in the chorus. Fourteen of the students sing in the chorus and also play in the school band. How many students on the school bus sing in the chorus but do not play in the band? 4



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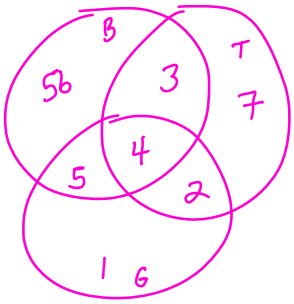


$$\begin{array}{r} 78 \\ -22 \\ \hline 56 \end{array}$$

Name: _____ Date: _____

A2CCH: Conditional Probability

When the probability of one event occurring changes depending on other events occurring then we say that there is a **conditional probability**. The language and symbolism of conditional probability can be a bit confusing, but the idea is fairly straightforward and can be developed with two-way frequency charts.

***Exercise* #1:** Let's revisit a two-way frequency chart we saw in the last lesson. In this study, 52 graduating seniors were surveyed as to their post-graduation plans and then the results were sorted by gender.

Let the following letters stand for the following events.

M = Male F = Female

C = Going to College N = Not going to college

	Gender		
	Male	Female	Total
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

If a person was picked at random, find the probability that the person was

(a) a female, i.e. $P(\mathrm{F}) = \frac{22}{52}$ or $\frac{11}{26}$

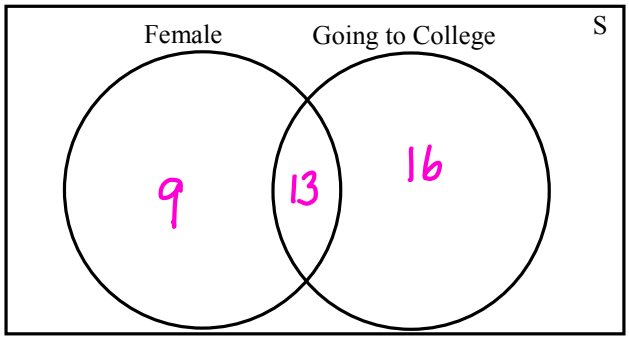
(b) going to college $P(\mathrm{C}) = \frac{29}{52}$

(c) going to college **given** they are female, i.e. $P(\mathrm{C} \mid \mathrm{F})$. Draw a Venn diagram below to help justify the ratio that you give as the probability.

"given"

$P(C|F) = \frac{n(C \text{ and } F)}{n(F)}$

$\frac{13}{22}$



(d) Which is more likely, that a person picked at random will be going to college, given they are a male, i.e. $P(\mathrm{C} \mid \mathrm{M})$, or that a person will be male, given they are going to college, i.e. $P(\mathrm{M} \mid \mathrm{C})$. Show that calculations for both.

$P(\mathrm{C} \mid \mathrm{M}) = \frac{n(C \text{ and } M)}{n(M)}$

$= \frac{16}{30}$

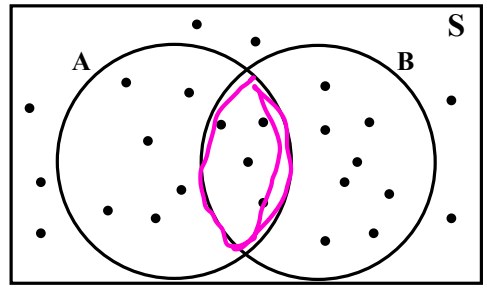
$P(\mathrm{M} \mid \mathrm{C}) = \frac{n(M \text{ and } C)}{n(C)}$

$= \frac{16}{29}$

more likely

We can generalize this process to calculate these conditional probabilities based on counts and a way to calculate these probabilities based on other probabilities.

***Exercise* #2:** In the generic Venn diagram shown to the right. Each dot represents an equally likely outcome of the sample space. Some of these fall only into event A, some only into event B, some in both events and some in neither.



(a) Consider the probability of A occurring given that B has occurred. Give a formula for this probability based on counting the number of elements in each set and their intersection.

$$P(\mathrm{A} \mid \mathrm{B}) = \frac{n(A \text{ and } B)}{n(B)} = \frac{4}{12} \text{ or } \frac{1}{3}$$

$$= \frac{P(A \text{ and } B)}{P(B)} = \frac{\frac{4}{25}}{\frac{12}{25}}$$

(b) Divide both of the numerator and denominator in (a) by the number of total elements in the sample space. Then rewrite the formula in (a) in terms of probabilities instead of counts.

$$P(\mathrm{A} \mid \mathrm{B}) = \frac{\frac{n(A \text{ and } B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \text{ and } B)}{P(B)}$$

$$\frac{4}{\cancel{25}} \cdot \frac{\cancel{25}}{12}$$

It's great when we can count elements that lie in events and their intersection, but sometimes we cannot. For example, let's revisit a relative frequency table that we saw in a previous homework.

***Exercise* #3:** A survey was taken to examine the relationship between hair color and eye color. The chart below shows the proportion of the people surveyed who fell into each category. If a person was picked at random, find each of the following conditional probabilities. Show the calculation you used.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

(a) Find the probability the person picked had brown eyes given they had blond hair.

$$P(\text{brown eyes} \mid \text{blond hair})$$

$$\frac{P(\text{brown eyes and blond hair})}{P(\text{blond hair})} = \frac{.10}{.35} = \frac{10}{35} \text{ or } \frac{2}{7}$$

(b) Find the probability the person had red hair given they had green eyes.

$$P(\text{red hair} \mid \text{green eyes}) = \frac{P(\text{red hair and green eyes})}{P(\text{green eyes})} = \frac{.15}{.25} = \frac{15}{25} = \frac{3}{5}$$

(c) Does having red hair seem have some **dependence** on having green eyes? How can you tell or quantify this dependence?

$P(\text{red hair}) = .20$

$P(\text{red} \mid \text{green}) = \frac{3}{5} = .60$ ← 3 times more likely to have red

Seems there is a dependence

Probability Rule Sheet

Never if you have green eyes

$$P(E) = \frac{n(E)}{n(S)}$$

$$0 \le P(E) \le 1$$

If A and B are complements:
$P(A) + P(B) = 1$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

"Probability of A given B"

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{or} \quad \frac{n(A \text{ and } B)}{n(B)}$$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{or} \quad \frac{n(A \text{ and } B)}{n(A)}$$

Name: Key Date: _____

A2CCH: Adding Probabilities

Homework 09-13

HOMEWORK

FLUENCY

1. Given the two sets below, give the sets that represent their union and their intersection.

$A = \{3, 5, 7, 9, 11, 13\}$ $B = \{1, 5, 9, 13, 17\}$

(a) Union: A or B =

$A \cup B = \{1, 3, 5, 7, 9, 11, 13, 17\}$

(b) Intersection: A and B =

$A \cap B = \{5, 9, 13\}$

2. Using sets A and B from #1, verify the addition law for the union of two sets:

$$n(\text{A or B}) = n(\text{A}) + n(\text{B}) - n(\text{A and B})$$

$6 + 5 - 3 = 8$

APPLICATIONS

3. Red Hook High School has 480 freshmen. Of those freshmen, 333 take Algebra, 306 take Biology, and 188 take both Algebra and Biology. Which of the following represents the number of freshmen who take at least one of these two classes?

$n(A \text{ or } B) = 333 + 306 - 188 = 451$

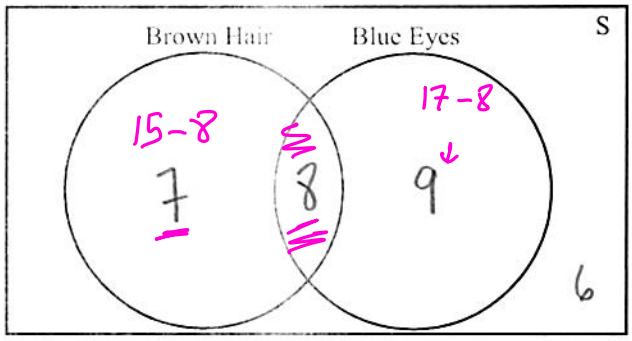
(1) 639 (3) 451

(2) 384 (4) 425

3

4. Evie was doing a science fair project by surveying her biology class. She found that of the 30 students in the class, 15 had brown hair and 17 had blue eyes and 6 had neither brown hair nor blue eyes. Determine the number of students who had brown hair and blue eyes. Use the Venn Diagram below to help sort the students if needed.

8



$30 - 6 = 24$

$15 + 17 = 32$

$32 - 24 = 8$

$15 + 17$
32

$30 - 6 = 24$

5. A standard six-sided die is rolled and its outcome noted. Which of the following is the probability that the outcome was less than three or even?

$P(L \cup E) = P(L) + P(E) - P(L \cap E)$

$\frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6}$

(1) $\frac{2}{3}$ (3) $\frac{5}{6}$

(2) $\frac{1}{3}$ (4) $\frac{1}{6}$

_____

6. Historically, a given day at the beginning of March in upstate New York has a 18% chance of snow and a 12% chance of rain. If there is a 4% chance it will rain and snow on a day, then which of the following represents the probability that a day in early March would have either rain or snow?

$P(S \text{ or } R) = .18 + .12 - .04$

$.30 - .04$

$.26$

(1) 0.30 (3) 0.02

(2) 0.34 (4) 0.26

4

7. A survey was done of students in a high school to see if there was a connection between a student's hair color and her or his eye color. If a student is chosen at random, find the probability of each of the following events.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

(a) The student had black hair. .45

(b) The student had blue eyes. .40

(c) The student had brown eyes and black hair. .25

(d) The student had blue eyes or blond hair.

$P(Blu \text{ or } Blo)$

$.4 + .35 - .20$

$.55$

(e) The student had black hair or blue eyes.

$P(Bla \cup Blue)$

$.45 + .40 - .15 = .70$

8. A recent survey of the Arlington High School 11th grade students found that 56% were female and 58% liked math as their favorite subject (of course). If 76% of all students are either female or liked math as their favorite subject, then what percent of the 11th graders were female students who liked math as their favorite subject? Show how you arrived at your answer.

F

M

$P(F \text{ or } M) = .76$

$P(F) = .56$

$P(M) = .58$

$P(F \text{ or } Math)$

$P(F \text{ or } M) = P(F) + P(Math) - P(F \text{ and } M)$

$.76 = .56 + .58 - P(Female \cap math)$

$.76 = 1.14 - P(F \cap m)$

$-.38 = -P(F \cap m)$

$P(F \cap m) = .38$

Name: Key

A2CCH: Venn Diagram Practice

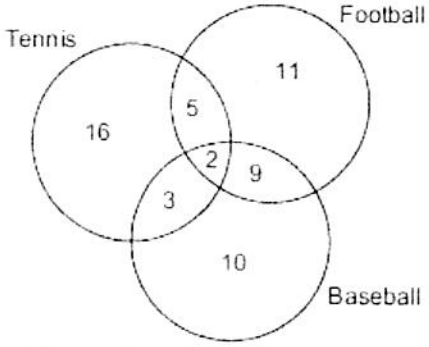
www.jmap.org

1 Monique has three sons who play football, two sons who play baseball, and one son who plays both sports. If all of her sons play baseball or football, how many sons does she have?

1) 5
2) 6
3) 3
4) 4

2 The accompanying diagram shows the results of a survey asking which sports the members of the Key Club watch on television.

Sports Watched on Television



F 27

B 24

Which statement or statements are true?

I The most watched sport is tennis. 26 ✗

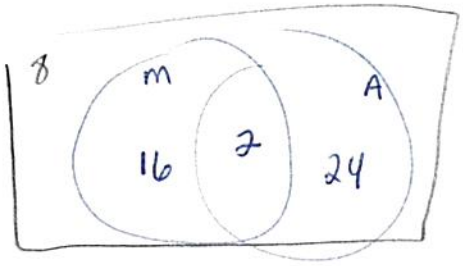
II The least watched sport is baseball. ✓

III More Key Club members watch tennis than football. ✗

1) I, only
2) II, only
3) I and II, only
4) II and III, only

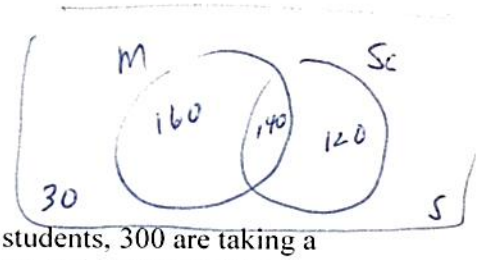
3 In a class of 50 students, 18 take music, 26 take art, and 2 take both art and music. How many students in the class are not enrolled in either music or art?

1) 6
2) 8
3) 16
4) 24



16+26 = 42

50 - 42



4 In a class of 450 students, 300 are taking a mathematics course and 260 are taking a science course. If 140 of these students are taking both courses, how many students are *not* taking either of these courses?

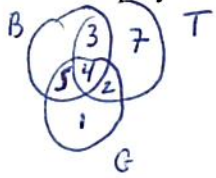
1) 30
2) 40
3) 110
4) 140

78

5 Seventy-eight students participate in one or more of three sports: baseball, tennis, and golf. Four students participate in all three sports; five play both baseball and golf, only; two play both tennis and golf, only; and three play both baseball and tennis, only. If seven students play only tennis and one plays only golf, what is the total number of students who play only baseball?

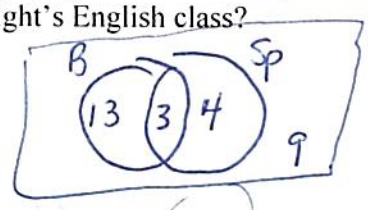
1) 12
2) 44
3) 56
4) 60

78 - 22



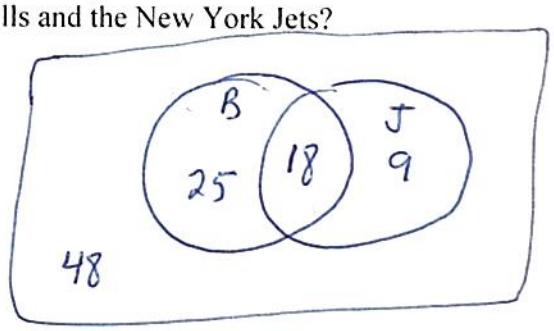
6 In Ms. Wright's English class, 16 students are in band, 7 students play sports, 3 students participate in both activities, and 9 students are not in band and do not play sports. How many students are in Ms. Wright's English class?

1) 10
2) 26
3) 29
4) 35



7 A school newspaper took a survey of 100 students. The results of the survey showed that 43 students are fans of the Buffalo Bills, 27 students are fans of the New York Jets, and 48 students do not like either team. How many of the students surveyed are fans of both the Buffalo Bills and the New York Jets?

1) 16
2) 18
3) 52



100 - 48 = 52