Name: _____ Date: _____

AP Calculus Properties of Definite Integrals Intro Ms. Loughran

Find each of the following integrals.

1. $\int_{2}^{2}(x^2+1)dx =$ 0

2. $\int_{-1}^{-1}(x-2)dx =$ 0

3. $\int_{-1}^{1}(x^2+1)dx =$ $\frac{8}{3}$

4. $\int_{1}^{2}(x^2+1)dx =$ $\frac{10}{3}$

5. $\int_{-1}^{2}(x^2+1)dx =$ 6

6. $\int_{2}^{-1}(x^2+1)dx =$ $-6$

7. $\int_{0}^{2}x^2dx =$ $\frac{8}{3}$

8. $\int_{-2}^{0}x^2dx =$ $\frac{8}{3}$

9. $\int_{-2}^{2}x^2dx =$ $\frac{16}{3}$

10. $\int_{0}^{2}x^3dx =$ 4

11. $\int_{-2}^{0}x^3dx =$ $-4$

12. $\int_{-2}^{2}x^3dx =$ 0

Now see if you can answer the following questions.

1. $\int_{a}^{a}f(x)dx =$ 0

2. Rewrite $\int_{a}^{b}f(x)dx+\int_{b}^{c}f(x)dx$ using a single integral. $\int_{a}^{c}f(x)dx$

3. What is the relationship between $\int_{a}^{b}f(x)dx$ and $\int_{b}^{a}f(x)dx$? $\int_{a}^{b}f(x)dx = -\int_{b}^{a}f(x)dx$

4. If $f(x)$ is an even function, fill in the following blank: $\int_{-a}^{a}f(x)dx =$ 2 $\int_{0}^{a}f(x)dx$

5. If $f(x)$ is an odd function, fill in the following blank: $\int_{-a}^{a}f(x)dx =$ 0

## USING INTEGRATION THEOREMS

**1.** Use the fact that

$$\int_0^2 x^2\,dx = \frac{8}{3}$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^2\,dx = \frac{8}{3}$ (b) $\int_{-2}^{2} x^2\,dx = 2\left(\frac{8}{3}\right) = \frac{16}{3}$

(c) $\int_0^2 -x^2\,dx$ $-\frac{8}{3}$ (d) $\int_0^2 (x^2+1)\,dx$

(e) $\int_{-2}^{0} 3x^2\,dx$

$\int_0^2 x^2\,dx + \int_0^2 1\,dx$

$\frac{8}{3} + x\Big|_0^2$

$\frac{8}{3} + (2-0) = \frac{14}{3}$

$-\int_0^2 x^2\,dx$

$3\int_{-2}^{0} x^2\,dx$

$3\left(\frac{8}{3}\right)$

$8$

**2.** Use the fact that

$$\int_0^2 x^3\,dx = 4$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^3\,dx = -4$ (b) $\int_{-2}^{2} x^3\,dx = 0$

(c) $\int_0^2 -x^3\,dx$ (d) $\int_0^2 (x^3+1)\,dx = \int_0^2 x^3\,dx + \int_0^2 1\,dx$

(e) $\int_{-2}^{0} 3x^3\,dx$

$4 + x\Big|_0^2$

$4 + (2-0) = 6$

c) $-\int_0^2 x^3\,dx = -4$

(e) $3\int_{-2}^{0} x^3\,dx$

$3(-4) = -12$

ROSLYN HIGH SCHOOL CALCULUS AB S. Conrad, S. Weiss

# ADDITIVITY

1. If $\int_0^5 f(x)\,dx = 10$ and $\int_5^7 f(x)\,dx = 3$, find

   (a) $\int_0^7 f(x)\,dx = 13$ (b) $\int_5^0 f(x)\,dx = -10$

   (c) $\int_5^5 f(x)\,dx = 0$ (d) $\int_0^5 3f(x)\,dx = 3(10) = 30$

2. If $\int_0^3 f(x)\,dx = 4$ and $\int_3^6 f(x)\,dx = -1$, find

   (a) $\int_0^6 f(x)\,dx = 3$ (b) $\int_6^3 f(x)\,dx$ 1

   (c) $\int_4^4 f(x)\,dx = 0$ (d) $\int_3^6 -5f(x)\,dx = -5(-1) = 5$

3. If $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$, find

   (a) $\int_2^6 [f(x) + g(x)]\,dx = \int_2^6 f(x)dx + \int_2^6 g(x)dx = 8$

   (b) $\int_2^6 [g(x) - f(x)]\,dx$ $-2 - 10 = -12$

   (c) $\int_2^6 [2f(x) - 3g(x)]\,dx$ $2(10) - 3(-2) = 26$

   (d) $\int_2^6 3f(x)\,dx = 3(10) = 30$

# ADDITIVITY OF THE INTEGRAL

(Abbreviated) Theorems: 1) $\int_a^b = \int_a^c + \int_c^b$ ; and 2) $\int_a^a = 0$; and 3) $\int_a^b = -\int_b^a$.

I: Assume that f has an integral on [1,7] some of whose values are given by: $\int_1^5 f(x)\,dx = 3$, $\int_2^3 f(x)\,dx = 1$, $\int_3^5 f(x)\,dx = 1$ and $\int_3^7 f(x)\,dx = 6$. Evaluate

ANSWERS

1) $\int_1^3 f(x)\,dx$

2) $\int_2^5 f(x)\,dx$

3) $\int_2^7 f(x)\,dx$

4) $\int_5^7 f(x)\,dx$

5) $\int_1^2 f(x)\,dx$

6) $\int_1^7 f(x)\,dx$

II: Assume that g has an integral on [1,9] some of whose values are given by: $\int_1^4 g(x)\,dx = 1$, $\int_2^4 g(r)\,dr = 2$, $\int_2^6 g(s)\,ds = 0$ and $\int_6^9 g(t)\,dt = 1$. Evaluate

ANSWERS

7) $\int_1^2 g(u)\,du$

8) $\int_2^9 g(v)\,dv$

9) $\int_4^6 g(w)\,dw$

10) $\int_1^9 g(x)\,dx$

11) $\int_4^9 g(y)\,dy$

12) $\int_1^6 g(z)\,dz$

13) $\int_6^4 g(t)\,dt$

1) $\int_1^3 f(x)\,dx$

$$\int_1^3 + \int_3^5 = \int_1^5$$

$$\int_1^3 = \int_1^5 - \int_3^5$$

$$= 3 - 1 = 2$$

2) $\int_2^5 f(x)\,dx$

$\int_2^3 + \int_3^5 =$ $1+1=2$

3) $\int_2^7 f(x)\,dx$

$\int_2^3 + \int_3^7 = 1+6=7$

# Homework 02-02

Name: _____ Date: _____

AP Calculus AB Evaluating Definite Integrals Ms. Loughran

**Part A. Directions:** Answer these questions *without* using your calculator.

**1.** $\int_{-1}^{1} (x^2 - x - 1)\,dx =$

**(A)** $\frac{2}{3}$ **(B)** 0 **(C)** $-\frac{4}{3}$ **(D)** $-2$ **(E)** $-1$

**2.** $\int_{1}^{2} \frac{3x-1}{3x}\,dx =$

**(A)** $\frac{3}{4}$ **(B)** $1 - \frac{1}{3}\ln 2$ **(C)** $1 - \ln 2$ **(D)** $-\frac{1}{3}\ln 2$ **(E)** 1

**3.** $\int_{0}^{3} \frac{dt}{\sqrt{4-t}} =$

**(A)** 1 **(B)** $-2$ **(C)** 4 **(D)** $-1$ **(E)** 2

**4.** $\int_{-1}^{0} \sqrt{3u+4}\,du =$

**(A)** 2 **(B)** $\frac{14}{9}$ **(C)** $\frac{14}{3}$ **(D)** 6 **(E)** $\frac{7}{2}$

**5.** $\int_{2}^{3} \frac{dy}{2y-3} =$

**(A)** $\ln 3$ **(B)** $\frac{1}{2}\ln\frac{3}{2}$ **(C)** $\frac{16}{9}$ **(D)** $\ln\sqrt{3}$ **(E)** $\sqrt{3} - 1$

**6.** $\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx =$

**(A)** 1 **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** $-1$ **(E)** 2

7. $\int_0^1 (2t-1)^3\,dt =$

(A) $\frac{1}{4}$ (B) 6 (C) $\frac{1}{2}$ (D) 0 (E) 4

8. $\int_4^9 \frac{2+x}{2\sqrt{x}}\,dx =$

(A) $\frac{25}{3}$ (B) $\frac{41}{3}$ (C) $\frac{100}{3}$ (D) $\frac{5}{3}$ (E) $\frac{1}{3}$

9. $\int_0^1 e^{-x}\,dx =$

(A) $\frac{1}{e} - 1$ (B) $1 - e$ (C) $-\frac{1}{e}$ (D) $1 - \frac{1}{e}$ (E) $\frac{1}{e}$

10. $\int_0^1 xe^{x^2}\,dx =$

(A) $e - 1$ (B) $\frac{1}{2}(e-1)$ (C) $2(e-1)$ (D) $\frac{e}{2}$ (E) $\frac{e}{2} - 1$

11. $\int_0^{\pi/4} \sin 2\theta\,d\theta =$

(A) 2 (B) $\frac{1}{2}$ (C) $-1$ (D) $-\frac{1}{2}$ (E) $-2$

12. $\int_1^2 \frac{dz}{3-z} =$

(A) $-\ln 2$ (B) $\frac{3}{4}$ (C) $2(\sqrt{2}-1)$ (D) $\frac{1}{2}\ln 2$ (E) $\ln 2$