USING INTEGRATION THEOREMS

1. Use the fact that

$$\int_0^2 x^2\,dx = \frac{8}{3}$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^2\,dx$ (b) $\int_{-2}^{2} x^2\,dx$

(c) $\int_0^2 -x^2\,dx$ (d) $\int_0^2 (x^2+1)\,dx$

(e) $\int_{-2}^{0} 3x^2\,dx$

2. Use the fact that

$$\int_0^2 x^3\,dx = 4$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^3\,dx$ (b) $\int_{-2}^{2} x^3\,dx$

(c) $\int_0^2 -x^3\,dx$ (d) $\int_0^2 (x^3+1)\,dx$

(e) $\int_{-2}^{0} 3x^3\,dx$

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ADDITIVITY

1. If $\int_0^5 f(x)\,dx = 10$ and $\int_5^7 f(x)\,dx = 3$, find

   (a) $\int_0^7 f(x)\,dx$ (b) $\int_5^0 f(x)\,dx$

   (c) $\int_5^5 f(x)\,dx$ (d) $\int_0^5 3f(x)\,dx$

2. If $\int_0^3 f(x)\,dx = 4$ and $\int_3^6 f(x)\,dx = -1$, find

   (a) $\int_0^6 f(x)\,dx$ (b) $\int_6^3 f(x)\,dx$

   (c) $\int_4^4 f(x)\,dx$ (d) $\int_3^6 -5f(x)\,dx$

3. If $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$, find

   (a) $\int_2^6 [f(x) + g(x)]\,dx$

   (b) $\int_2^6 [g(x) - f(x)]\,dx$

   (c) $\int_2^6 [2f(x) - 3g(x)]\,dx$

   (d) $\int_2^6 3f(x)\,dx$

ADDITIVITY OF THE INTEGRAL

(Abbreviated) Theorems: 1) $\int_a^b = \int_a^c + \int_c^b$; and 2) $\int_a^a = 0$; and 3) $\int_a^b = -\int_b^a$.

I: Assume that f has an integral on [1,7] some of whose values are given by: $\int_1^5 f(x)\,dx = 3$, $\int_2^3 f(x)\,dx = 1$, $\int_3^5 f(x)\,dx = 1$ and $\int_3^7 f(x)\,dx = 6$. Evaluate

ANSWERS

1) $\int_1^3 f(x)\,dx$

2) $\int_2^5 f(x)\,dx$

3) $\int_2^7 f(x)\,dx$

4) $\int_5^7 f(x)\,dx$

5) $\int_1^2 f(x)\,dx$

6) $\int_1^7 f(x)\,dx$

II: Assume that g has an integral on [1,9] some of whose values are given by: $\int_1^4 g(x)\,dx = 1$, $\int_2^4 g(r)\,dr = 2$, $\int_2^6 g(s)\,ds = 0$ and $\int_6^9 g(t)\,dt = 1$. Evaluate

ANSWERS

7) $\int_1^2 g(u)\,du$

8) $\int_2^9 g(v)\,dv$

9) $\int_4^6 g(w)\,dw$

10) $\int_1^9 g(x)\,dx$

11) $\int_4^9 g(y)\,dy$

12) $\int_1^6 g(z)\,dz$

13) $\int_6^4 g(t)\,dt$