Name: ________________________________ Date: _________

AP Calculus AB Ms. Loughran

Do Now:

Evaluate.

1. $\lim_{x\to\infty}\frac{\sqrt{x}-4}{4-3\sqrt{x}}$ $= -\frac{1}{3}$

2. If $y=\sin^3(1-2x)$, find y'.

$y' = 3\sin^2(1-2x)\cdot\cos(1-2x)\cdot -2$

$y' = -6\sin^2(1-2x)\cos(1-2x)$

3. If $y=x^2e^{\frac{1}{x}}$, find $\frac{dy}{dx}$ $= x^2e^{\frac{1}{x}}\cdot -x^{-2} + 2xe^{\frac{1}{x}}$

$-\frac{\cancel{x^2}e^{\frac{1}{x}}}{\cancel{x^2}} + 2xe^{\frac{1}{x}}$

$-e^{\frac{1}{x}} + 2xe^{\frac{1}{x}}$

$e^{\frac{1}{x}}(-1+2x)$

3. The table at the right records the values of f, g, f', and g' at x = 1 and x = 2. Find the value of each of the following:

x	f(x)	g(x)	f'(x)	g'(x)
1	2	2	4	5
2	5	7	9	3

a) $(f + g)'(2)$

$f'(2) + g'(2)$

$9 + 3 = 12$

b) $\left(\frac{f}{g}\right)'(1)$

$\dfrac{gf' - fg'}{g^2}$

$\dfrac{g(1)f'(1) - f(1)g'(1)}{g^2(1)} = \dfrac{(2)(4) - (2)(5)}{2^2} = \dfrac{-2}{4} = -\dfrac{1}{2}$

c) $\frac{d}{dx}f[g(x)]$ at $x = 1$

$f'(g(x)) \cdot g'(x)$

$f'(g(1)) \cdot g'(1)$

$f'(2) \cdot g'(1)$

$9 \cdot 5 = 45$

d) $(g \circ f)'(1)$ $= \left(g(f(x))\right)'$ at $x=1$

$g'(f(1)) \cdot f'(1)$

$g'(2) \cdot f'(1)$

$3 \cdot 4 = 12$

e) $[f^2(x)]'$ at $x = 2$

$2f(x) \cdot f'(x)$

$2f(2) \cdot f'(2)$

$10 \cdot 9$ $\quad 90$

f) $[f(x^2)]'$ at $x = 1$

$f'(x^2) \cdot 2x$

$f'(1^2) \cdot 2(1)$

$4 \cdot 2$

8

g) $(f \circ f)'(1)$

$f'(f(1)) \cdot f'(1)$

$f'(2)$

$9 \cdot 4 = 36$

4. Determine the derivative g'(x) in terms of f' if

a) $g(x) = f(x^2)$

$$g'(x) = f'(x) \cdot 2x$$

b) $g(x) = f(\sin^2 x) + f(\cos^2 x)$

$$g'(x) = f'(\sin^2 x) \cdot \underbrace{2\sin x \cos x}_{\sin 2x} + f'(\cos^2 x) \cdot \underbrace{2\cos x(-\sin x)}_{-\sin 2x}$$

$$g'(x) = f'(\sin^2 x)\sin 2x - f'(\cos^2 x)\sin 2x$$

c) $g(x) = f[f(x)]$

$$g'(x) = f'(f(x)) \cdot f'(x)$$

d) $g(x) = f(f[f(x)])$

$$g'(x) = f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x)$$

Homework 10-16

Name: Key
AP Calc: Derivatives of e^u and $\ln u$ Homework

Date: ___________
Ms. Loughran

For each of the following, find $\frac{dy}{dx}$.

1. $y = \ln 2x$

$y' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

2. $y = x^3 \ln x$

$y' = x^3 \cdot \frac{1}{x} + 3x^2 \ln x = x^2 + 3x^2 \ln x$

3. $y = \sqrt{\ln x} = (\ln x)^{\frac{1}{2}}$

$y' = \frac{1}{2}(\ln x)^{-\frac{1}{2}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$

4. $y = \cos(\ln x)$

$y' = -\sin(\ln x) \cdot \frac{1}{x} = \frac{-\sin(\ln x)}{x}$

5. $y = x^3 e^x$

$y' = x^3 e^x + 3x^2 e^x = x^2 e^x (x + 3)$

6. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$= \frac{4}{(e^x + e^{-x})^2}$ or $\frac{4e^{2x}}{e^{4x} + 2e^{2x} + 1}$

7. $y = e^{x \tan x}$

$y' = e^{x \tan x} \cdot (x \sec^2 x + \tan x)$

8. $y = e^{(x - e^{3x})}$

$y' = e^{x - e^{3x}} \cdot (1 - 3e^{3x})$

9. $y = \ln(1 - xe^{-x})$

$y' = \frac{1}{1 - xe^{-x}} \cdot (-(-xe^{-x} + e^{-x}))$

$y' = \frac{xe^{-x} - e^{-x}}{1 - xe^{-x}} = \frac{\frac{x}{e^x} - \frac{1}{e^x}}{1 - x\frac{1}{e^x}} \cdot \frac{e^x}{e^x} = \frac{x - 1}{e^x - x}$

$$(e^{-x})' = e^{-x} \cdot -1$$

6. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$y' = \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

$$y' = \frac{\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - (\cancel{e^{2x}} - 2 + \cancel{e^{-2x}})}{(e^x + e^{-x})^2}$$

$$y' = \frac{4}{(e^x + e^{-x})^2} \quad \text{or} \quad \frac{4}{e^{2x} + 2 + e^{-2x}} = \frac{4}{e^{2x} + 2 + \frac{1}{e^{2x}}} = \frac{4e^{2x}}{e^{4x} + 2e^{2x} + 1}$$

6. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$= \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

continue from there