Name:______________________________ Date:__________

AP Calc AB Intro to Max-Min Problems Ms. Loughran

$A = lw$

1. My dachshund, Daisie, needs a place to play. I purchased 100 feet of fencing to use to make her an enclosed rectangular play area. What should the dimensions of this play area be in order for her to have the largest play area possible? What is the resulting maximum area?

$50 - x$

$\frac{100-2x}{2}$

restrictions:

$x > 0$ $\quad 50 - x > 0$, $50 > x$

$0 < x < 50$

x $\quad x$

25 ft by 25 ft

max area: $625\ \text{ft}^2$

$A(x) = x(50 - x) = 50x - x^2$

$A'(x) = 50 - 2x$

$50 - 2x = 0$

$x = 25$

SDT: $A''(x) = -2$

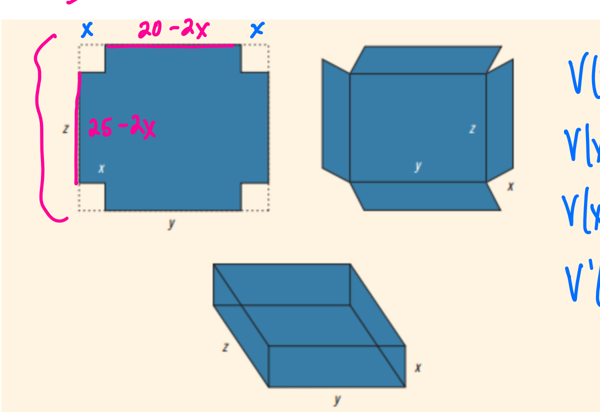
$A''(x) < 0$

$\cap$ $A(x)$ CD so there is a max at $x = 25$

2. Now that Daisie has a place to play, she needs to have a place to keep her toys. I want to make an open-top box so that she can reach in and get the toys out herself. I want to use a 20-by-25 inch sheet of tin to make this box. I will cut congruent squares of side length *x* from the corners of the sheet of tin and bend up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting volume?

3.681 in

$V(3.681) = 820.528\ \text{in}^3$



$V = lwh$

$V(x) = (20-2x)(25-2x)x$

$V(x) = (500 - 90x + 4x^2)x$

$V(x) = 500x - 90x^2 + 4x^3$

$V'(x) = 12x^2 - 180x + 500$

$0 = 12x^2 - 180x + 500$

$0 = 3x^2 - 45x + 125$

$x = \frac{45 \pm \sqrt{(-45)^2 - 4(3)(125)}}{2(3)} = \frac{45 \pm \sqrt{525}}{6}$

$\frac{45 + \sqrt{525}}{6} = 11.318$ outside the restriction

$\frac{45 - \sqrt{525}}{6} = 3.681$

$V''(x) = 24x - 180$

$V''(3.681) < 0$

$\cap$ there is a max at 3.681

Restrictions: $0 < x < 10$

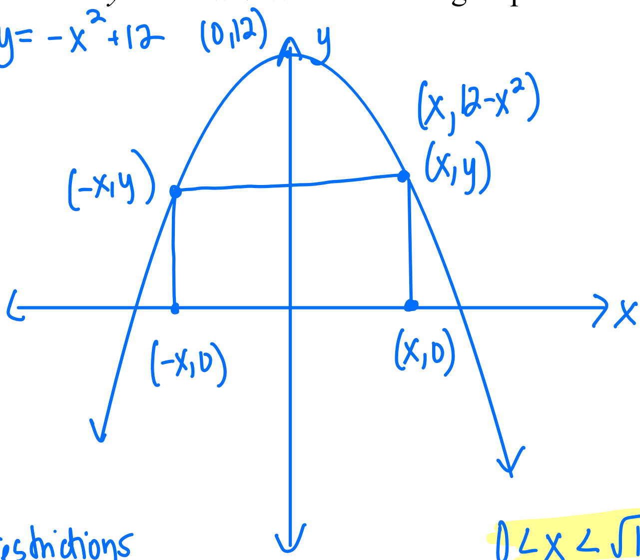
$x > 0$

$20 - 2x > 0$, $20 > 2x$, $10 > x$, $x < 10$

$25 - 2x > 0$, $25 > 2x$, $x < \frac{25}{2}$

3. A rectangle has its base on the x-axis and its two upper corners on the parabola $y = 12 - x^2$. What is the largest possible area of the rectangle?

$y = -x^2 + 12$



$A = lw$

$A(x) = 2x(12 - x^2)$

$A(x) = 24x - 2x^3$

$A'(x) = 24 - 6x^2$

$24 - 6x^2 = 0$

$24 = 6x^2$

$4 = x^2$

$x = \cancel{\pm}2$

b/c it is outside our restrictions

$A''(x) = -12x$

$A''(2) < 0$

there is a max at $x=2$

$A(x)$

$A(2) = 24(2) - 2(2)^3$

$A(2) = 48 - 16$

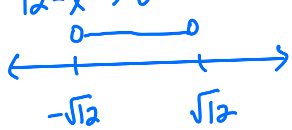
$A(2) = 32$ units^2

restrictions

$2x > 0$

$x > 0$

$12 - x^2 > 0$



$0 < x < \sqrt{12}$

or

$0 < x < 2\sqrt{3}$

4. An open rectangular box is to be made from a 9 by 12 inch piece of tin by cutting squares of side x inches from the corners and folding up the sides. What should x be to maximize the volume of the box?

5. A 384-square-meter plot of land is to be enclosed by a fence and divided into two equal parts by another fence parallel to one pair of sides. What dimensions of the outer rectangle will minimize the amount of fence used?

6. What is the radius of a cylindrical soda can with volume of 512 cubic inches that will use the minimum material?

$$SA_{can} = 2\pi r^2 + 2\pi r h$$

* need h in terms of r

$$V = \pi r^2 h$$

$$512 = \pi r^2 h$$

$$\frac{512}{\pi r^2} = h$$

restrictions

$r > 0$

If $r > 0$, automatically $h > 0$

$$SA(r) = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{512}{\cancel{\pi} r \cancel{2}}\right)$$

$$SA(r) = 2\pi r^2 + 1024 r^{-1}$$

$$SA'(r) = 4\pi r - 1024 r^{-2}$$

$$0 = 4\pi r - \frac{1024}{r^2}$$

$$\frac{1024}{r^2} = 4\pi r$$

$$4\pi r^3 = 1024$$

$$r^3 = \frac{1024}{4\pi}$$

$$r^3 = \frac{256}{\pi}$$

$$r = \sqrt[3]{\frac{256}{\pi}}$$

Homework 12-21

Name: KEY

AP Calc AB: More Local Linear Approximations

Date:__________

Ms. Loughran

$f(x)=\sqrt[3]{x}$ (64,4) pt

$y-4=\frac{1}{48}(x-64)$

$y-4=\frac{1}{48}(63-64)$

$y-4=-\frac{1}{48}$

$y=3\frac{47}{48}$

(64,4)

$y=x^{\frac{1}{3}}$

$y'=\frac{1}{3}x^{-2/3}$

$y'\big|_{64}\ \frac{1}{3}(64)^{-2/3}=\frac{1}{48}$

1. Approximately how much less than 4 is $\sqrt[3]{63}$?

   (a) $\frac{1}{48}$ (b) $\frac{1}{16}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ (e) 1

2. The best linear approximation for $f(x)=\tan x$ near $x=\frac{\pi}{4}$ is

   (a) $1+\frac{1}{2}(x-\frac{\pi}{4})$ (b) $1+(x-\frac{\pi}{4})$ (c) $1+\sqrt{2}(x-\frac{\pi}{4})$

   (d) $1+2(x-\frac{\pi}{4})$ (e) $2+2(x-\frac{\pi}{4})$

   $f'(x)=\sec^2 x$

   $f'(\pi/4)=\sec^2(\pi/4)=2$

   $y-1=2(x-\pi/4)$

   $y=2(x-\pi/4)+1$

Can change h to x if its easier to work with

3. When h is near zero, e^{kh}, using local linearization, is approximately

   (a) k (b) kh (c) 1 (d) $1+k$ (e) $1+kh$

if $h=0$ (0,1) (h,y) or let $h=x$

$y=e^{kh}$

$y'=e^{kh}\cdot k$

$y'\big|_{h=0}=k$

tan line

$y-1=k(x-0)$

$y-1=k(0-0)$

$y-1=0$

$y=1$

$h=0$

$y=e^{kh}$ $y=1$

(0,1)

$y'=e^{kh}\cdot k$

$y'\big|_{h=0}=1\cdot k$

tan. line

$y-1=k(h-0)$

when $h=0$

$y-1=k(0-0)$

$y-1=0$

$y=1$

4. If $f(6)=30$ and $f'(x)=\frac{x^2}{x+3}$, then an estimate of $f(6.02)$, using the local linearization, is

   (a) 29.92 (b) 30.02 (c) 30.08 (d) 34.00 (e) none of these

   (6,30)

   $f'(6)=\frac{6^2}{6+3}=4$

   $y-30=4(x-6)$

   $y-30=4(6.02-6)$ .02

   $y-30=.08$

   $y=30.08$

5. The tangent line approximation for $f(x)=\sqrt{x^2+16}$ near $x=-3$ is

   (a) $5-\frac{3}{5}(x-3)$ (b) $5+\frac{3}{5}(x-3)$ (c) $5-\frac{3}{5}(x+3)$

   (d) $3-\frac{5}{3}(x-3)$ (e) $3+\frac{3}{5}(x+3)$

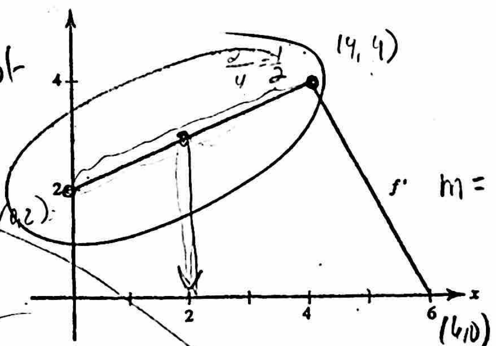
   $f(x)=(x^2+16)^{1/2}$

   $f'(x)=\frac{1}{2}(x^2+16)^{-1/2}\cdot 2x=\frac{x}{\sqrt{x^2+16}}\Big|_{x=-3}$ $-3/5$

   $y-5=-\frac{3}{5}(x+3)$

   $y=-3/5(x+3)+5$

6. The graph of f' is shown below. If we know that $f(2)=10$, then the local linearization of f at $x=2$ is $f(x)=$



Find eq. of line

(2,10)

$\boxed{f'(2)=3}$

$y-10=3(x-2)$

$y=3x-6+10$

$m=\frac{4-0}{4-6}=\frac{4}{-2}=-2$

$y=-2(x-6)$

$m=\frac{4-2}{4-0}=\frac{2}{4}=\frac{1}{2}$

$y-2=\frac{1}{2}(x)$

$y=\frac{1}{2}x+2$

$y=\frac{1}{2}(2)+2=3$

   (a) $\frac{x}{2}+2$ (b) $\frac{x}{2}+9$ (c) $3x-3$ (d) $3x+4$ (e) $10x-17$

piecewise function

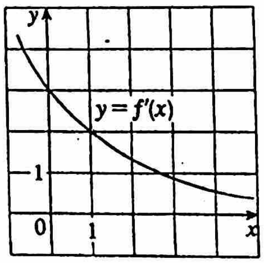
$$f'(x)=\begin{cases}\frac{1}{2}x+2 & 0\le x\le 4\\ -2x+12 & 4<x\le 6\end{cases}$$

7. Suppose that the only information we have about a function, f, is that $f(1) = 5$ and the graph of its derivative is as shown.
   (a) Use a linear approximation to estimate $f(0.9)$ and $f(1.1)$.
   (b) Are your estimates in part (a) too large or too small? Explain.

$(1,5)$

$f'(1) = 2$

(a) $y - 5 = 2(x-1)$



$f' \downarrow$ so

f'' ⊖.

f'' ⊖.

$f(0.9) = 2(-.1) + 5 = 4.8$

$f(1.1) = 2(.1) + 5 = 5.2$

(b) f is concave down so tangent line is above x so estimates are too big, overestimates

8. Suppose that we don't have a formula for $g(x)$ but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2 + 5}$ for all x.
   (a) Use a linear approximation to estimate $g(1.95)$ and $g(2.05)$.
   (b) Are your estimates in part (a) too large or too small? Explain.

$g'(2) = \sqrt{2^2 + 5} = 3$

$y + 4 = 3(x-2)$

$g''(x) = \frac{1}{2}(x^2+5)^{-\frac{1}{2}} \cdot 2x$

$g''(x) = \frac{x}{\sqrt{x^2+5}}$

(a) $g(1.95) = 3(\underbrace{1.95-2}_{-.05}) - 4 = -4.15$

$g(2.05) = 3(\underbrace{2.05-2}_{.05}) - 4 = -3.85$

g'': $-$ | 0 | $+$ ∪

g'' is + for $x > 0$ so g is concave up to the right of zero and therefore at $x = 2$ so the tangent lines lie below $g(x)$ so the estimations are too small.