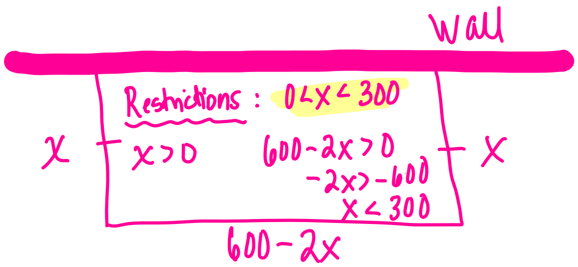
Do Now: #1

Name: ______________________________ Date: ________

AP Calculus AB More Max-Min Problems (Sheet 3)

1. A farmer has 600 m of fencing with which he plans to enclose a rectangular pen adjacent to a long existing wall. He will use the wall for one side of the pen and the available fencing for the three remaining sides. What is the maximum area that he can enclose this way?



$A = lw$

$A(x) = x(600-2x)$

$A(x) = 600x - 2x^2$

$A'(x) = 600 - 4x$

$600 - 4x = 0$

$600 = 4x$

$x = 150$

$A(150) = 150(600-2(150))\,m^2$

$= 45000\,m^2$

$A''(x) = -4$

$A''(150) < 0$

at 150 there is a max

2. The sum of two positive numbers is 48. What is the smallest possible value of the sum of their squares?

x = one positive #

$48-x$ = other positive #

restrictions $x>0$ $\quad 0<x<48$

$48-x>0$

$48>x$

$x<48$

$S(x) = x^2 + (48-x)^2$

$S(x) = x^2 + x^2 - 96x + 2304$

$S(x) = 2x^2 - 96x + 2304$

$S'(x) = 4x - 96$

$4x - 96 = 0$

$x = 24$

$S''(x) = 4$

$S''(24) > 0$

$24^2 + 24^2 = 1152$

* If the problem read as two non negative numbers, our restrictions would be $0 \le x \le 48$ we would have to consider $S(0)$, $S(48)$

3. A rectangular box has a square base whose edge is at least 1 cm, and its total surface area is 600 cm^2. What is the largest possible volume that such a box can have?

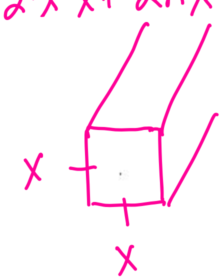
$SA = 2lw + 2hw + 2lh$

$600 = 2\cdot x\cdot x + 2hx + 2xh$

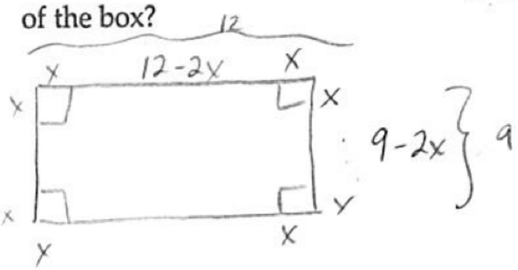
$V = lwh$

$V(x) = x\cdot x\cdot h$

* need h in terms of x



4. An open rectangular box is to be made from a 9×12 inch piece of tin by cutting squares of side x inches from the corners and folding up the sides. What should x be to maximize the volume of the box?



$$V(x) = x(12-2x)(9-2x)$$
$$V(x) = x(108 - 42x + 4x^2)$$
$$V(x) = 108x - 42x^2 + 4x^3$$

$$V'(x) = 108 - 84x + 12x^2$$
$$V'(x) = 12(9 - 7x + x^2)$$
$$x^2 - 7x + 9 = 0$$
$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(9)}}{2(1)}$$
$$x = \frac{7 \pm \sqrt{13}}{2}$$

5.303 reject outside rest.

1.697

$V'(x)$: $+$ | 1.697 | $-$

$V(x)$: $\nearrow$ $\searrow$

So we have a rel max at

$$x = \frac{7 - \sqrt{13}}{2} \text{ in or } 1.697 \text{ in}$$

Rest.

$$9 - 2x > 0$$
$$-2x > -9$$
$$x < \frac{9}{2}$$

$$12 - 2x > 0$$
$$-2x > -12$$
$$x < 6$$

$$x > 0$$

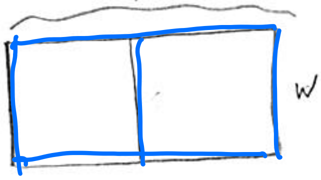
$$\therefore \; 0 < x < \frac{9}{2}$$

$$V''(x) = -84 + 24x$$
$$V''(1.697) < 0$$

$A = lw$ $\quad 384 = lw$ $\quad \frac{384}{l} = w$

5\. A 384-square-meter plot of land is to be enclosed by a fence and divided into two equal parts by another fence parallel to one pair of sides. What dimensions of the outer rectangle will minimize the amount of fence used?

$384/w$



$w > 0$

$\frac{384}{w} > 0$

$A = 384 \text{ m}^2$

$lw = 384 \text{ m}^2$

$l = \frac{384}{w}$

$\text{Fence} = 3w + 2(384/w)$

$= 3w + 768/w$

$= 3w + 768w^{-1}$

$F' = 3 - 768w^{-2}$

$0 = 3 - \frac{768}{w^2}$

$3 = \frac{768}{w^2}$

$3w^2 = 768$

$w^2 = 256$

$w = \cancel{\pm}16$

$w = 16 \text{ m}$

$l = \frac{384}{16} = 24 \text{ m}$

$w = -16$ is outside of our restriction set

checking it using 2nd derivative test

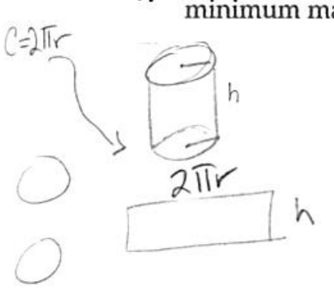
$F'' = \frac{1536}{w^3}$

$F''(16) > 0$ so we have rel min at $w = 16$

*may want to use 2nd derivative test b/c plugging into 1st derivative is annoying here

6\. What is the radius of a cylindrical soda can with volume of 512 cubic inches that will use the minimum material?

Need h in terms of r



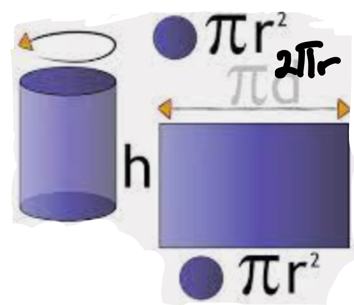
$V = \pi r^2 h$

$512 = \pi r^2 h$

$\frac{512}{\pi r^2} = h$

$\frac{512}{\pi r^2} > 0$

$r > 0$



$r > 0$

$SA(r) = 2\pi r h + 2\pi r^2$

$SA(r) = 2\cancel{\pi r}\left(\frac{512}{\cancel{\pi} r^{\cancel{2}}}\right) + 2\pi r^2$

$SA(r) = 1024r^{-1} + 2\pi r^2$

$SA'(r) = -1024r^{-2} + 4\pi r$

$0 = \frac{-1024}{r^2} + 4\pi r$

$4\pi r = \frac{1024}{r^2}$

$4\pi r^3 = 1024$

$\pi r^3 = 256$

$r^3 = \frac{256}{\pi}$

$\Rightarrow r = \sqrt[3]{\frac{256}{\pi}} \text{ in}$

$SA''(r) = 2048r^{-3} + 4\pi$

$SA''\left(\sqrt[3]{\frac{256}{\pi}}\right) = \frac{2048}{\left(\sqrt[3]{\frac{256}{\pi}}\right)^3} + 4\pi > 0$

so we have a rel min at

$r = \sqrt[3]{\frac{256}{\pi}}$ in.

Again you may want to use 2nd derivative test here bc plugging into 1st derivative is annoying + too hard w/ calculator