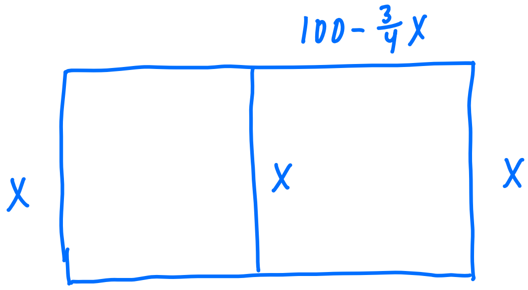
Name: ___________________________________ Date: ___________

AP Calculus AB: More Optimization Problems Ms. Loughran

Do Now : #s 1 and 2

1. A rancher wants to construct two identical rectangular corrals using 400 feet of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?



$\frac{400-3x}{4}$

$A = lw$

$A(x) = x\left(100-\frac{3}{4}x\right)$

$A(x) = 100x - \frac{3}{4}x^2$

$A'(x) = 100 - \frac{3}{2}x$

Rest: $x > 0$

$100 - \frac{3}{4}x > 0$

$-\frac{3}{4}x > -100$

$x < \frac{400}{3}$

Rest: $\left(0, \frac{400}{3}\right)$

$100 - \frac{3}{2}x = 0$

$100 = \frac{3}{2}x$

$\frac{200}{3} = x$

$A''(x) = -\frac{3}{2}$

there is a max at $\frac{200}{3}$

$\frac{200}{3}$ ft by $100 - \frac{3}{4}\left(\frac{200}{3}\right)$ ft

2. Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 500 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

since it is open top 1(lw)

$SA = 2lw + 2lh + 2wh$

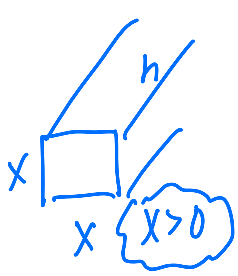
$SA = x^2 + 4xh$ *need h in terms of x

$V = 500 ft^3$

$500 = lwh$

$500 = x^2 h$

$\frac{500}{x^2} = h$



$SA(x) = x^2 + 4x\left(\frac{500}{x^2}\right)$

$SA(x) = x^2 + 2000x^{-1}$

$SA'(x) = 2x - 2000x^{-2}$

$SA''(x) = 2 + 4000x^{-3}$ ∴ min at x=10

$2x = \frac{2000}{x^2}$

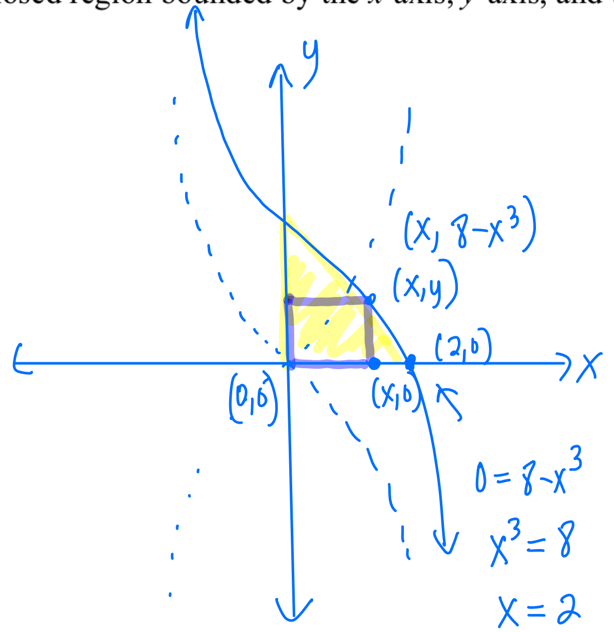
$2x^3 = 2000$

$x^3 = 1000$

$x = 10$

10ft × 10ft × $\frac{500}{10^2}$ ft or 5ft

6. Find the dimensions of the rectangle of largest area which can be inscribed in the closed region bounded by the x-axis, y-axis, and the graph of $y = 8 - x^3$.



$y = -x^3 + 8$

x^3

reflected over x-axis ↑8

$0 = 8 - x^3$

$x^3 = 8$

$x = 2$

$A = lw$

$A(x) = x(8 - x^3)$

$A(x) = 8x - x^4$

$A'(x) = 8 - 4x^3$

Restrictions

$x > 0$

$x < 2$

$0 < x < 2$

$8 - 4x^3 = 0$

$8 = 4x^3$

$2 = x^3$

$\sqrt[3]{2} = x$

$A''(x) = -12x^2 < 0$

∴ at $\sqrt[3]{2}$ there is a max

$\sqrt[3]{2}$ by $8 - (\sqrt[3]{2})^3$

$\sqrt[3]{2}$ by 6

Homework 01-03

2. The sum of two positive numbers is 48. What is the smallest possible value of the sum of their squares?

$x = \#$

$48 - x = $ other #

rest.

$0 < x < 48$

If they said two nonnegative #s $0 \le x \le 48$ and then since it would be a closed interval we would have to check * for abs max/min

$x^2 + (48-x)^2 = S$

$x^2 + 2304 - 96x + x^2 = S$

$2x^2 - 96x + 2304 = S$

$S' = 4x - 96$

$4x - 96 = 0$

$4x = 96$

$x = 24$

S': $-$ | $+$ at 24

S: $\searrow$ 24 $\nearrow$

So we have a rel min at $x = 24$

$S(24) = 1152$

$* \; S(0) = 2304$

$* \; S(48) = 2304$

$S''(x) = 4$ $\cup$

3. A rectangular box has a square base whose edge is at least 1 cm, and its total surface area is 600 cm^2. What is the largest possible volume that such a box can have?

If $x = \sqrt{300}$, $\sqrt{300} \cdot \sqrt{300} \cdot \frac{300 - (\sqrt{300})^2}{2(\sqrt{300})}$

$1 \le x < \sqrt{300}$ ← not an endpoint

$\frac{300 - x^2}{2x} > 0$

$300 - x^2 > 0$

$-\sqrt{300}$ $\sqrt{300}$

$V(10) = 1000$ cm^3

$V(1) = (1)(1)\left(\frac{299}{2}\right) = \frac{299}{2}$ cm^3

$V(\sqrt{300}) = (\sqrt{300})(\sqrt{300})(0) = 0$

$SA = 2lw + 2hw + 2lh$

$SA = 2x \cdot x + 2hx + 2xh$

$SA = 2x^2 + 2hx + 2hx$

$SA = 2x^2 + 4hx$

$600 = 2x^2 + 4hx$

$\frac{600 - 2x^2}{4x} = \frac{4hx}{4x}$

$\frac{300 - x^2}{2x} = h$

$V(x) = lwh$

$V(x) = x \cdot x \cdot \frac{300 - x^2}{2x}$

$V(x) = \frac{x(300 - x^2)}{2} = \frac{1}{2}(300x - x^3)$

$V'(x) = \frac{1}{2}(300 - 3x^2)$

$0 = \frac{1}{2}(300 - 3x^2)$

$0 = 300 - 3x^2$

$3x^2 = 300$

$x^2 = 100$

$x = \pm 10$

$V''(x) = \frac{1}{2}(-6x)$

$V''(x) = -3x$

$V''(10) < 0$ $\cap$ at $x = 10$

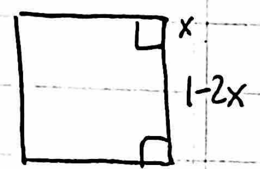
$V'(x)$: $-$ | $+$ | $-$ at -10, 10; min at -10, max at 10

$h = \frac{300 - 100}{20} = \frac{200}{20} = 10$ cm

$w = l = 10$ cm

④

$$V(x) = 3\left[x(1-2x)(1-2x)\right] + 2x^3$$



$$V(x) = 3x - 12x^2 + 12x^3 + 2x^3$$

$$V(x) = 14x^3 - 12x^2 + 3x$$

$x > 0$

$1 - 2x > 0$

$x < \frac{1}{2}$

$0 < x < \frac{1}{2}$

$$V'(x) = 42x^2 - 24x + 3$$

$$0 = 42x^2 - 24x + 3$$

$$0 = 3(14x^2 - 8x + 1)$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(14)(1)}}{2(14)}$$

$$x = \frac{8 \pm \sqrt{8}}{28} \quad .387 \quad .185$$

$$V''(x) = 84x - 24$$

$V''(.387) > 0$ ∴ @ .387 we have rel. min

$V''(.185) < 0$ ∴ @ .185 we have rel max

The boxes should be .185m x .185m

(5) $SA = 2\pi r^2 + 2\pi rh$

$*$ Need h in terms of r

$r > 0$

$\frac{1000}{\pi r^2} > 0$

$V = \pi r^2 h$

$1000 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

$SA(r) = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right)$

$SA(r) = 2\pi r^2 + 2000\, r^{-1}$

$SA'(r) = 4\pi r - \frac{2000}{r^2}$

$4\pi r - \frac{2000}{r^2} = 0$

$4\pi r = \frac{2000}{r^2}$

$4\pi r^3 = 2000$

$\pi r^3 = 500$

$r^3 = \frac{500}{\pi}$

$r = \sqrt[3]{\frac{500}{\pi}} \approx 5.419$

SA′: $-$ | $+$ at 5.419

SA: $\searrow$ | $\nearrow$

rel. min.

$\boxed{\begin{array}{l} r = 5.419 \text{ cm} \\ h = 10.840 \text{ cm} \end{array}}$

or

$r = \sqrt[3]{\frac{500}{\pi}}$ cm

$h = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2}$ cm

$SA''(r) = 4\pi + \frac{4000}{r^3}$

$SA''\left(\sqrt[3]{\frac{500}{\pi}}\right) > 0$ $\therefore$ at $r = \sqrt[3]{\frac{500}{\pi}}$ there is a minumum

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2. Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 500 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

3. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $\$10 / ft^2$ and the material used to build the sides cost $\$6 / ft^2$. If the box must have a volume of $50 ft^3$ determine the dimensions that will minimize the cost to build the box.

4. Engineers are constructing an open rectangular box with a square base from $48 ft^2$ of material. What dimensions will result in a box with the largest possible volume?

5. We want to construct a container in the shape of a right circular cylinder with no top and a surface area of $3\pi ft^2$. What height, *h*, and base radius, *r*, will maximize the volume of the cylinder?

6. Find the dimensions of the rectangle of largest area which can be inscribed in the closed region bounded by the *x*-axis, *y*-axis, and the graph of $y = 8 - x^3$.

7. A sheet of cardboard 3 feet by 4 feet will be made into a box by cutting equal sized squares from each corner and folding up the four edges. What would the dimensions of the box with the largest volume be?