Name: ______________________________ Date: __________

AP Calculus AB Ms. Loughran

1. Let f be the function defined by $f(x) = xe^{1-x}$ for all real numbers x.

   (a) Find each interval on which f is increasing.

   (b) Find the x-coordinate of each point of inflection of f.

   (c) Using the results found in parts (a) and (b), sketch the graph of f in the xy-plane.

a) $f'(x) = e^{1-x} + x \cdot e^{1-x} \cdot -1$

$f'(x) = e^{1-x} - xe^{1-x}$

$f'(x) = e^{1-x}(1-x)$

$e^{1-x}(1-x) = 0$

$e^{1-x} \neq 0 \quad | \quad 1-x=0$

$x = 1$

a) $(-\infty, 1)$

f': + | 1 | −

f: ↗ ↘

$f''(x) = e^{1-x}(-1) + (1-x)e^{1-x}(-1)$

$f''(x) = -e^{1-x}(1+1-x)$

b) $x=2$

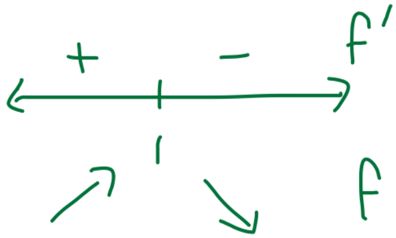
$f''(x) = -e^{1-x}(2-x)$

$-e^{1-x}(2-x) = 0$

$\emptyset \quad | \quad x=2$

f'': − | 2 | +

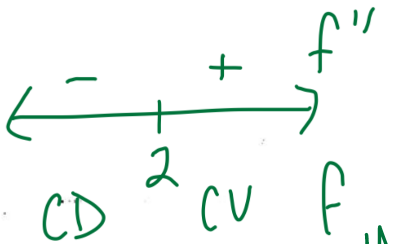
f: CD CV



$f(x) = xe^{1-x}$

$0 = x(e^{1-x})$

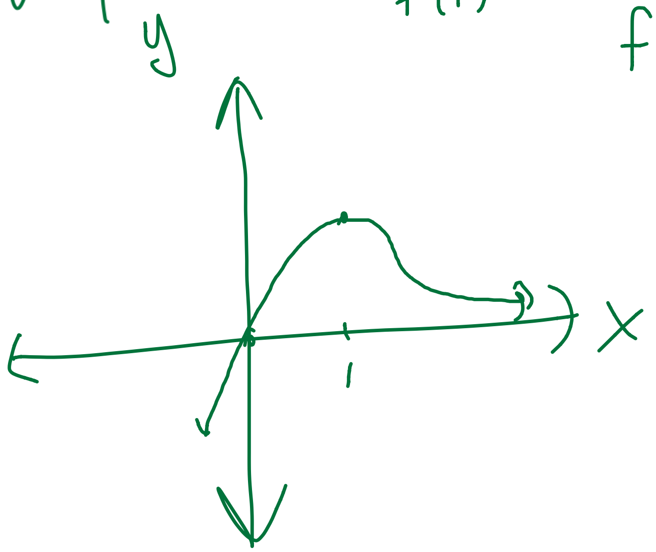
$x = 0$ | $\cancel{0}$ $(0,0)$



$f(1) = 1 \cdot e^{1-1} = 1$ $(1,1)$

$f(2) = 2e^{-1} = \frac{2}{e}$

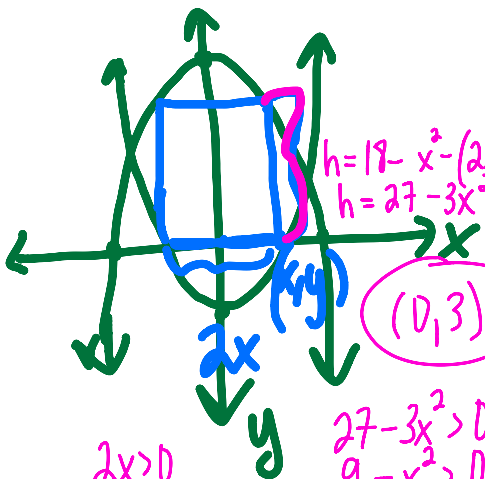
$f(3) = 3e^{-2}$



* $f(x) > 0$ when $x > 0$

1971- AB 4

Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of $f(x) = 18 - x^2$ and $g(x) = 2x^2 - 9$.



$h = 18 - x^2 - (2x^2 - 9)$

$h = 27 - 3x^2$

(0,3)

$f(x) = 18 - x^2$

$f(0) = 18$

$18 - x^2 = 0$

$18 = x^2$

$\pm\sqrt{18} = x$

$\pm 3\sqrt{2} = x$

$2x > 0$

$x > 0$

$27 - 3x^2 > 0$

$9 - x^2 > 0$

$-3 \quad + \quad 3 \quad -$

$A(x) = 2x(27 - 3x^2)$

$A(x) = 54x - 6x^3$

$A'(x) = 54 - 18x^2$

$54 - 18x^2 = 0$

$54 = 18x^2$

$3 = x^2$

$\pm\sqrt{3} = x$

$A''(x) = -36x$

$A''(\sqrt{3}) < 0$ $\cap$ ← MAX

$f(x) = 2x^2 - 9$

$f(0) = -9$

$2x^2 - 9 = 0$

$2x^2 = 9$

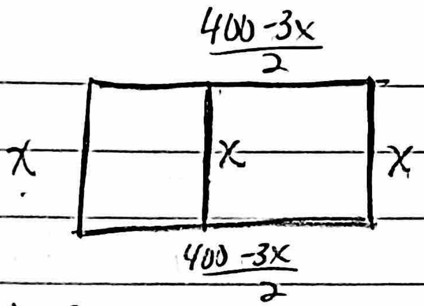
$x^2 = \pm\frac{3}{\sqrt{2}}$

$A(\sqrt{3}) = 54\sqrt{3} - 6(\sqrt{3})^3$

Key to More Optimization

$2 \cdot \frac{400-3x}{4} = \frac{400-3x}{2}$ ①

①



$\frac{400-3x}{2}$

$x > 0$

$\frac{400-3x}{2} > 0$

$200 - \frac{3}{2}x > 0$

$-\frac{3}{2}x > -200$

$\frac{3}{2}x < 200$

$3x < 400$

$x < \frac{400}{3}$

$A = lw$

$A(x) = (200 - \frac{3}{2}x)x$

$A(x) = 200x - \frac{3}{2}x^2$

$A'(x) = 200 - 3x$

$200 - 3x = 0$

$200 = 3x$

$x = \frac{200}{3}$

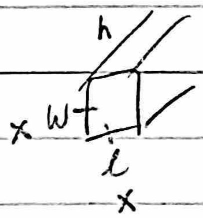
$A''(x) = -3$

$A''(\frac{200}{3}) = -3 < 0$ ∴ at $x = \frac{200}{3}$ there is a max

$\frac{200}{3}$ ft by $\frac{400 - 3(\frac{200}{3})}{2} = \frac{200}{2} = 100$

$\frac{200}{3}$ ft by 100 ft or $\frac{200}{3}$ ft by 50 ft per corral

②



bottom is a square

open top

$V = lwh$ $x > 0$

$500 = x \cdot x \cdot h$

$\frac{500}{x^2} = h$

$SA = lw + 2lh + 2wh$

$SA(x) = x^2 + 2x \cdot \frac{500}{x^2} + 2x \cdot \frac{500}{x^2}$

$SA(x) = x^2 + 2000x^{-1}$

$SA'(x) = 2x - 2000x^{-2}$

10 by 10 by 5 ft

$2x - \frac{2000}{x^2} = 0$

$2x = \frac{2000}{x^2}$

$2x^3 = 2000$

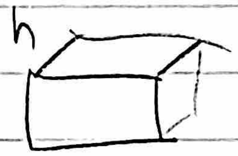
$x^3 = 1000$

$x = 10$

$SA''(x) = 2 + 4000x^{-3}$ ∴ there is a min at $x = 10$

$SA''(10) > 0$

(3)



$$V = lwh$$
$$50 = x \cdot 3x \cdot h$$
$$50 = 3x^2 h$$
$$\frac{50}{3x^2} = h$$

$x > 0$

bases

$$SA = 2lw + 2lh + 2wh$$
$$Cost(x) = 10 \cdot 2 \cdot 3x \cdot x + 6 \cdot 2 \cdot \cancel{3x} \cdot \frac{50}{\cancel{3}x^{\cancel{2}}} + \overset{2}{\cancel{6}} \cdot 2 \cdot \cancel{x} \cdot \frac{50}{\cancel{3}x^{\cancel{2}}}$$

$$Cost(x) = 60x^2 + 600x^{-1} + 200x^{-1}$$
$$Cost(x) = 60x^2 + 800x^{-1}$$
$$Cost'(x) = 120x - 800x^{-2}$$

$$120x - \frac{800}{x^2} = 0$$
$$120x = \frac{800}{x^2}$$
$$120x^3 = 800$$
$$x^3 = \frac{800}{120}$$
$$x^3 = 20/3$$
$$x = \sqrt[3]{20/3}$$

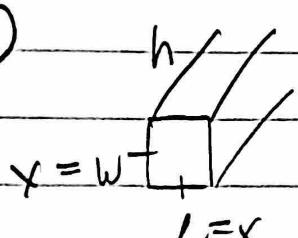
$$Cost''(x) = 120 + 1600x^{-3}$$
$$Cost''(\sqrt[3]{20/3}) > 0 \quad \cup$$

$\therefore$ there is a min at $\sqrt[3]{\frac{20}{3}}$ ft

$\sqrt[3]{20/3}$ ft by $3\sqrt[3]{20/3}$ ft by $\frac{50}{3\left(\sqrt[3]{\frac{20}{3}}\right)^2}$

* open top *

4



$SA = lw + 2lh + 2wh$

$SA(x) = x^2 + 2xh + 2xh$ $\qquad x > 0$

$SA(x) = x^2 + 4xh$

$48 = x^2 + 4xh$ $\qquad 48 - x^2 > 0$

$\frac{48 - x^2}{4x} = h$

$-4\sqrt{3}$ $\qquad 4\sqrt{3}$

$0 < x < 4\sqrt{3}$

$V(x) = x \cdot \cancel{x} \cdot \frac{48 - x^2}{4\cancel{x}} = \frac{48x - x^3}{4} = 12x - \frac{1}{4}x^3$

$V'(x) = 12 - \frac{3}{4}x^2$

$V''(x) = -\frac{6}{4}x$

$12 - \frac{3}{4}x = 0$ $\qquad V''(4) = \frac{-6}{4} \cdot 4 = -6 < 0$

$12 = \frac{3}{4}x^2$ $\qquad \therefore$ at $x = 4$

$48 = 3x^2$ $\qquad$ there is a rel

$x^2 = 16$ $\qquad$ max

$x = \pm 4$ reject -4 outside of interval

4 ft by 4 ft by $\frac{48 - 4^2}{4(4)}$ (16)

$\frac{32}{16}$

4 ft by 4 ft by 2 ft

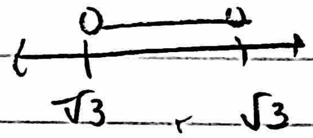
5

$SA = \pi r^2 + 2\pi rh$ $\qquad r > 0$

$3\cancel{\pi} = \cancel{\pi}r^2 + 2\cancel{\pi}rh$

$3 = r^2 + 2rh$ $\qquad 3 - r^2 > 0$

$\frac{3-r^2}{2r} = \frac{\cancel{2r}h}{\cancel{2r}}$



$V = \pi r^2 h$ $\qquad \therefore 0 < r < \sqrt{3}$

$V(r) = \pi r^{\cancel{2}} \cdot \frac{3-r^2}{\cancel{2r}} = \frac{\pi}{2}(3r - r^3)$

$V'(r) = \frac{\pi}{2}(3 - 3r^2)$

$V''(r) = \frac{\pi}{2}(-6r)$

$\frac{\pi}{2}(3 - 3r^2) = 0$ $\qquad V''(1) < 0 \quad \cap \quad \therefore$ there is a max at $r = 1$

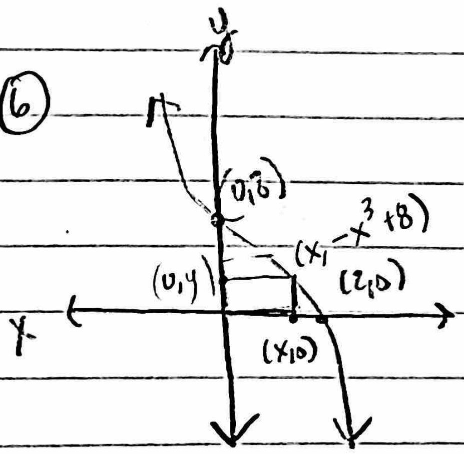
$3 - 3r^2 = 0$ $\qquad$ height 1 ft

$3 = 3r^2$ $\qquad$ radius = 1 ft

$1 = r^2$

$r = \pm 1$ reject -1

6



$y = 8 - x^3$ $\qquad$ x-intercept : $0 = -x^3 + 8$

$y = -x^3 + 8$ $\qquad x^3 = 8$

x^3 reflected over x-axis ↑8 $\qquad x = 2$

$A(x) = x(-x^3 + 8)$ $\qquad 0 < x < 2$

$A(x) = -x^4 + 8x$

$A'(x) = -4x^3 + 8$

↓

$-4x^3+8=0$

$-4x^3=-8$

$x^3=2$

$x=\sqrt[3]{2}$

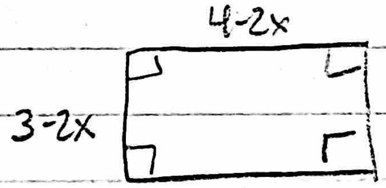
$A''(x)=-12x^2$

$A''(\sqrt[3]{2})<0 \ \cap \ \therefore$ there is a max at $x=\sqrt[3]{2}$

$\sqrt[3]{2}$ by $8-(\sqrt[3]{2})^3$

$\sqrt[3]{2}$ by 6

7



$V(x)=x(3-2x)(4-2x)$

$V(x)=(3x-2x^2)(4-2x)$

$V(x)=12x-6x^2-8x^2+4x^3$

$V(x)=4x^3-14x^2+12x$

$V'(x)=12x^2-28x+12$

$12x^2-28x+12=0$

$4(3x^2-7x+3)=0$

$x=\dfrac{7\pm\sqrt{(7)^2-4(3)(3)}}{2(3)}=\dfrac{7\pm\sqrt{13}}{6}$ reject $\dfrac{7+\sqrt{13}}{6}$

$x>0$

$3-2x>0$ $\quad$ $4-2x>0$

$-2x>-3$ $\quad$ $-2x>-4$

$x<3/2$ $\quad$ $x<2$

$0<x<3/2$

$V''(x)=24x-28$

$V''\left(\frac{7-\sqrt{13}}{6}\right)=24\left(\frac{7-\sqrt{13}}{6}\right)-28=28-4\sqrt{13}-28<0 \ \cap$

$\therefore$ there is a max at $\dfrac{7-\sqrt{13}}{6}$

$\left(3-2\left(\frac{7-\sqrt{13}}{6}\right)\right)$ ft by $\left(4-2\left(\frac{7-\sqrt{13}}{6}\right)\right)$ ft by $\dfrac{7-\sqrt{13}}{6}$