Do Now: #4 from yesterday's sheet

Check that the hypotheses of the Mean-Value Theorem are satisfied on the given interval. If so, find all the values of c in that interval that satisfy the conclusion of the theorem.

4. $f(x) = 2x^{\frac{1}{4}}$; $[-2,1]$

$f(x) = 2\sqrt[4]{x}$

$f'(x) = \frac{1}{2}x^{-\frac{3}{4}} = \frac{1}{2\sqrt[4]{x^3}}$

↑ 0 is problematic so it not diff on $(-2,1)$

cont: $[-2,1]$ we could argue that since it is defined for $x \geq 0$ we would argue that it is continuous on $[-2,1]$

MTV doesn't apply

$$f'(x) = \begin{cases} 2 & x \le -1 \\ 3x^2 - 1 & x > -1 \end{cases}$$

5. If $f(x) = \begin{cases} 2x+2, & \text{if } x \le -1 \\ x^3 - x, & \text{if } x > -1 \end{cases}$, determine whether f satisfies the conditions of the mean value theorem on $[-3,2]$. If the function does satisfy the conditions of the mean value theorem, find the value of c.

cont on $[-3,2]$ ✓

$$\lim_{x\to -1^-} f(x) = \lim_{x \to -1^+} f(x)$$

$$2(-1)+2 = (-1)^3 - (-1)$$

$$0 = 0 \checkmark$$

diff. $(-3,2)$ ✓

$$\lim_{x\to -1^-} f'(x) = \lim_{x\to -1^+} f'(x)$$

$$2 = 3(-1)^2 - 1 \checkmark$$

average rate of change:

$$\frac{f(2)-f(-3)}{2-(-3)} = \frac{6-(-4)}{5} = 2$$

$x \le -1$

$$2 = 2$$

always true for

$c \le -1$

$x > -1$

$$2 = 3c^2 - 1$$

$$3 = 3c^2$$

$$1 = c^2$$

$$\pm 1 = c$$

we have to fit these inside our interval $(-3, 2)$

$(-3,-1] \cup \{1\}$

Complete #s 1-10

What You Need to Know About Motion Along the x-axis (Part 1)

In discussing motion, there are three closely related concepts that you need to keep straight. These are:

$x(t)$ or $s(t) \to$ position

$x'(t)$ or $s'(t) = v(t) \to$ velocity

$x''(t)$ or $s''(t) = v'(t) \to$ acceleration

If $x(t)$ represents the position of a particle along the x-axis at any time t, then the following statements are true.

1. "Initially" means when time, t = 0.
2. "At the origin" means $x(t)$, position = 0.
3. "At rest" means $v(t)$ = 0.
4. If the velocity of the particle is positive, then the particle is moving to the right.
5. If the velocity of the particle is negative, then the particle is moving to the left.
6. To find average velocity over a time interval, divide the change in $x(t)$ by the change in time.
7. Instantaneous velocity is the velocity at a single moment (instant!) in time.
8. If the acceleration of the particle is positive, then the velocity is increasing.
9. If the acceleration of the particle is negative, then the velocity is decreasing.
10. In order for a particle to change direction, the velocity must change signs.
11. One way to determine total distance traveled over a time interval is to find the sum of the absolute values of the differences in position between all resting points. Here's an example: If the position of a particle is given by:

$$x(t) = \frac{1}{3}t^3 - t^2 - 3t + 4,$$

find the total distance traveled on the interval $0 \le t \le 6$.

$v(t) = t^2 - 2t - 3$

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0$

$t = 3, \cancel{-1}$

$x(0) = 4$ } 9

$x(3) = -5$ } 27

$x(6) = 22$

TD: 36 units

$\cancel{9} - \cancel{9} - 9 + 4$

$72 - 36 - 18 + 4$

displacement: $22 - 4 = 18$

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Example 1 (analytic).

A particle moves along the x-axis so that at time t its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

$v(t) = 3t^2 - 12t + 9$

$v(0) = 9$

right b/c $v(0) > 0$

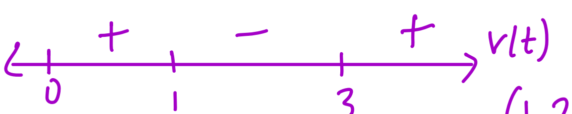
2. At $t = 1$, is the velocity of the particle increasing or decreasing? Explain your answer.

$v'(t) = a(t) = 6t - 12$

$a(1) = -6$

decreasing b/c $v'(1)$ or $a(1) < 0$

3. Find all values of t for which the particle is moving to the left.



$(1, 3)$

$3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 1, 3$

4. Find the total distance traveled by the particle over the time interval $0 \le t \le 5$.

$x(0) = 11$
$x(1) = 15$ } 4
$x(3) = 11$ } 4
$x(5) = 31$ } 20

TD: 28 units

② displacement: $31 - 11 = 20$ units

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Example 2 (analytic)

A particle is moving along a horizontal line according to the function:

$$S(t) = \frac{t^3}{3} - 3t^2 + 8t + 4 \qquad \text{for } t \geq 0$$

where t is time in seconds and s is measured in feet.

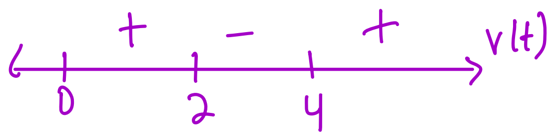
a) At t = 1, is the particle moving to the right or left? Explain.

$v(t) = t^2 - 6t + 8$

$v(1) = 3$

right b/c $v(1) > 0$

b) During what time interval(s) is the particle moving to the left? Explain.



$t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

$t = 2, 4$

$(2,4)$ b/c $v(t) < 0$ in that interval

c) What is the average velocity from t=0 to t= 3 sec?

$$\frac{s(3) - s(0)}{3-0} = \frac{10-4}{3} = 2 \text{ ft/s}$$

~~d)~~ Is the particle speeding up or slowing down at t=1? Explain.

e) What is the total distance traveled from t=0 to t=4 sec?

$s(0) = 4 = \frac{12}{3}$

$s(2) = \frac{32}{3}$

$s(4) = \frac{28}{3}$

$\frac{20}{3}$

$\frac{4}{3}$

TD: $\frac{24}{3} = 8$ units

3

displacement $= \frac{28}{3} - \frac{12}{3} = \frac{16}{3}$

Homework 01-08

* Check that final c is in the interval given *

(12) $f'(x) = x^3 + x - 4 \quad [-1,2]$

continuous on $[-1,2]$
diff at $(-1,2)$

$f'(x) = 3x^2 + 1$

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$3c^2 + 1 = \dfrac{f(2) - f(-1)}{2-(-1)}$

$3c^2 + 1 = \dfrac{6-(-6)}{3}$

$3c^2 + 1 = 4$

$3c^2 = 3$

$c^2 = 1$

$c = \pm 1$ reject $c=-1$ b/c not in $(-1,2)$

* can't be at an endpoint

$c = 1$ is in $(-1,2)$

(14) $f(x) = x + \frac{1}{x} \quad [3,4] \quad f(x) = x + x^{-1}$

continuous on $[3,4]$ + diff on $(3,4)$

$f'(x) = 1 - \frac{1}{x^2}$

$1 - \frac{1}{c^2} = \dfrac{\frac{17}{4} - \frac{10}{3}}{4-3}$

$1 - \frac{1}{c^2} = \frac{11}{12}$

$-\frac{1}{c^2} = -\frac{1}{12}$

$\frac{1}{c^2} = \frac{1}{12}$

$c^2 = 12$

$c = \pm 2\sqrt{3}$

$c = 2\sqrt{3}$ is in $(3,4)$

(16) $f(x) = \frac{1}{x-1} \; ; \; [2,5]$

$(x-1)^{-1}$

$f'(x) = -(x-1)^{-2}$

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\dfrac{-1}{(c-1)^2} = \dfrac{\frac{1}{4} - 1}{5-2}$

$\dfrac{-1}{(c-1)^2} = \dfrac{-3/4}{3}$

$\dfrac{-1}{(c-1)^2} = -\frac{1}{4}$

$(c-1)^2 = 4$

$c - 1 = \pm 2$

$c = 1 \pm 2$

$c = 3, -1$ reject

$c = 3$

Investigating Rolle's Thm

(19) $f(x) = \tan x$

$f'(x) = \sec^2 x$

$0 = \sec^2 x$

has no solution

$\sec x = 0$

$\sec^{-1}(0) = \emptyset$

$\cos^{-1}$ (undefined) $\emptyset$

(b) $\tan x$ is not continuous on $[0, \pi]$

(23) $s(t)$ is the position fn of automobile for $0 \le t \le 5$

$$s'(c) = v(c) = \frac{s(5) - s(0)}{5 - 0} = \frac{4}{5} = .8 \text{ mi/min}$$

$$.8 \text{ mi/min} \cdot 60 \text{ min/hr} = 48 \text{ mi/hr}$$

(20) $f(x) = x^{2/3}$, $a = -1$, $b = 8$

$f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

(b) not differentiable on $(-1, 8)$

$$\frac{2}{3\sqrt[3]{c}} = \frac{4-1}{8-(-1)}$$

$$\frac{2}{3\sqrt[3]{c}} = \frac{3}{9}$$

$$\frac{2}{3\sqrt[3]{c}} = \frac{1}{3}$$

$3\sqrt[3]{c} = 6$

$\sqrt[3]{c} = 2$

$c = 2^3 = 8$

8 is not in $(-1, 8)$ open interval.

(b) not differentiable at $x = 0$ which is in $(-1, 8)$