Worksheet 3. Understanding the Relationships Among Velocity, Speed, and Acceleration

Speed is the absolute value of velocity. It tells you how fast something is moving without regard to the direction of movement.

1. What effect does absolute value have on numbers?

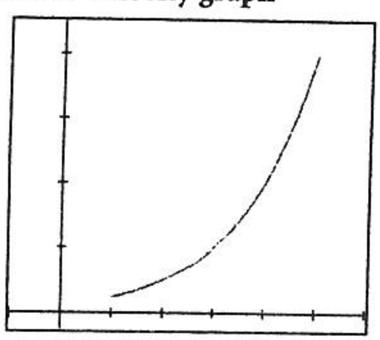
Absolute value makes all #s non negative

2. What effect does taking the absolute value of a function have on its graph?

Any portion of the graph that is below the x-axis gets reflected over the x-axis.

For each situation below, the graph of a differentiable function giving velocity as a function of time t is shown for $1 \le t \le 5$, along with selected values of the velocity function. In the graph, each horizontal grid mark represents 1 unit of time and each vertical grid mark represents 4 units of velocity. For each situation, plot the speed graph on the same coordinate plane as the velocity graph and fill in the speed values in the table. Then, answer the questions below based on both the graph and the table of values.

Situation 1: Velocity graph



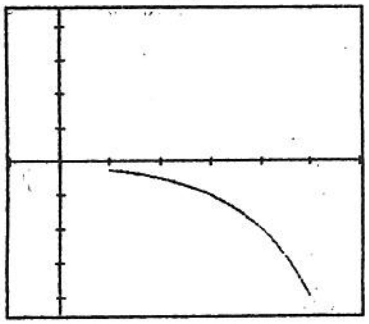
time	velocity	speed $\lvert v \rvert$
1	1	1
2	2	2
3	4	4
4	8	8
5	16	16

In this situation, the velocity is ______________ and ______________.
Positive or negative? Increasing or decreasing?

Because velocity is ______________, we know acceleration is ______________.
Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is ______________.
Increasing or decreasing?

Situation 2: Velocity graph



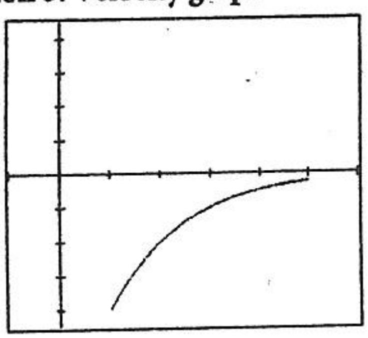
time	velocity	speed
1	-1	1
2	-2	2
3	-4	4
4	-8	8
5	-16	16

In this situation, the velocity is ______________ and ______________.
Positive or negative? Increasing or decreasing?

Because velocity is ______________, we know acceleration is ______________.
Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is ______________.
Increasing or decreasing?

Situation 3: Velocity graph



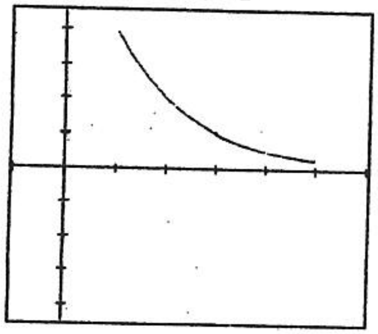
time	velocity	speed
1	-16	16
2	-8	8
3	-4	4
4	-2	2
5	-1	1

In this situation, the velocity is ______________ and ______________.
Positive or negative? Increasing or decreasing?

Because velocity is ______________, we know acceleration is ______________.
Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is ______________.
Increasing or decreasing?

Situation 4: Velocity graph



time	velocity	speed
1	16	16
2	8	8
3	4	4
4	2	2
5	1	1

In this situation, the velocity is ____________ and ____________.
Positive or negative? Increasing or decreasing?

Because velocity is ____________, we know acceleration is ____________
Increasing or decreasing? Positive or negative?

By examining the graph of speed and the table of values, we can conclude that speed is ____________
Increasing or decreasing?

Conclusion:

In which situations was the speed increasing? 1 + 2

When the speed is increasing, the velocity and acceleration have ____________ signs.
Same or opposite?

In which situations was the speed decreasing? 3 + 4

When the speed is decreasing, the velocity and acceleration have ____________ signs.
Same or opposite?

Dixie Ross, Pflugerville High School, Pflugerville, Texas

Assessing Students' Understanding (A Short Quiz):

1. If velocity is negative and acceleration is positive, then speed is decreasing.

2. If velocity is positive and speed is decreasing, then acceleration is negative.

3. If velocity is positive and decreasing, then speed is decreasing.
   acc ⊖

4. If speed is increasing and acceleration is negative, then velocity is ⊖.

5. If velocity is negative and increasing, then speed is decreasing.
   acc +

6. If the particle is moving to the left and speed is decreasing, then acceleration is
   vel ⊖
   positive.

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(11) $s(t) = t^3 - 6t^2, \quad t \geq 0$

a) $v(t) = 3t^2 - 12t$
$a(t) = 6t - 12$

b) $s(1) = 1^3 - 6(1)^2 = 1 - 6 = -5$ ft
$v(1) = 3(1)^2 - 12(1) = -9$ ft/s
speed$(1) = 9$ ft/s
$a(1) = 6(1) - 12 = -6$ ft/s^2

c) $v(t) = 0$
$3t^2 - 12t = 0$
$3t(t-4) = 0$
$t = 0s \quad t = 4s$

d)
$v(t)$: $-$ on $(0, 4)$, $+$ after 4

$a(t)$: $-$ on $(0, 2)$, $+$ after 2

* when acceleration is 0 speed is constant

$6t - 12 = 0$
$t = 2$

Speeding ↑: $(0,2) \cup (4, \infty)$
Slowing ↓: $(2,4)$

e) $s(0) = 0$
$s(4) = -32$
$s(5) = -25$

$32 + 7 = 39$ ft

(12) $s(t) = t^4 - 4t + 2$

a) $v(t) = 4t^3 - 4$
$a(t) = 12t^2$

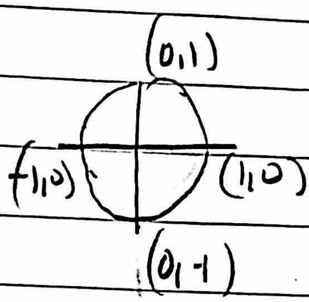
b) $s(1) = 1 - 4 + 2 = -1$ ft
$v(1) = 4(1)^3 - 4 = 0$ ft/s
speed$(1) = 0$ ft/s
$a(1) = 12$ ft/s^2

c) $4t^3 - 4 = 0$
$4(t^3 - 1) = 0$
$4(t-1)(t^2+t+1) = 0$
$t = 1s \quad | \quad t = \frac{-1 \pm \sqrt{1-4}}{2}$
imaginary

e) $s(0) = 2$
$s(1) = -1$
$s(5) = 607$

$3 + 608 = 611$ ft

(13) $s(t) = 3\cos\left(\frac{\pi}{2}t\right)$



a) $v(t) = -3\sin\left(\frac{\pi}{2}t\right) \cdot \frac{\pi}{2} = -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}t\right)$

$a(t) = -\frac{3\pi}{2}\cos\left(\frac{\pi}{2}t\right) \cdot \frac{\pi}{2} = \frac{-3\pi^2}{4}\cos\left(\frac{\pi}{2}t\right)$

b) $s(1) = 3\cos\left(\frac{\pi}{2}(1)\right) = 3\cos\left(\frac{\pi}{2}\right) = 3(0) = 0$ ft

$v(1) = -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}(1)\right) = -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}\right) = -3\pi/2$ ft/s

speed(1) $= 3\pi/2$ ft/s

$a(1) = \frac{-3\pi^2}{4}\cos\left(\frac{\pi}{2}(1)\right) = \frac{-3\pi^2}{4}\cos\left(\frac{\pi}{2}\right) = \frac{-3\pi^2}{4}(0) = 0$ ft/s^2

c) $v(t) = 0$

$\frac{2}{3\pi} \cdot -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}t\right) = 0 \cdot \frac{-2}{3\pi}$

$\sin\left(\frac{\pi}{2}t\right) = 0$

$\frac{\pi}{2}t = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$

$\frac{\pi}{2}t = 0$	$\frac{\pi}{2}t = \pi$	$\frac{\pi}{2}t = 2\pi$	$\frac{\pi}{2}t = 3\pi$
$t = 0$	$\frac{1}{2}t = 1$	$\frac{1}{2}t = 2$	$\frac{1}{2}t = 3$
	$t = 2$	$t = 4$	$\cancel{t = 6}$

outside dof $0 \le t \le 5$

$t = 0, 2, 4$ sec

(13) (e) $s(0) = 3\cos(\frac{\pi}{2}\cdot 0) = 3\cos(0) = 3$

$s(2) = 3\cos(\frac{\pi}{2}\cdot 2) = -3$

$s(4) = 3\cos(\frac{\pi}{2}\cdot 4) = 3\cos(2\pi) = 3$

$s(5) = 3\cos(\frac{\pi}{2}\cdot 5) = 3\cos(\frac{5\pi}{2}) = 0$

6

6

3

15 ft

(14) $s(t) = \frac{t}{t^2+4}$

(a)
$$v(t) = \frac{(t^2+4)(1) - t(2t)}{(t^2+4)^2} = \frac{4-t^2}{(t^2+4)^2}$$

$-2t^2$

$$a(t) = \frac{(t^2+4)^2(-2t) - (4-t^2)\cdot 2(t^2+4)\cdot 2t}{((t^2+4)^2)^2}$$

$$a(t) = \frac{-2t(t^2+4)^2 - 4t(t^2+4)(4-t^2)}{(t^2+4)^4}$$ * factor out GCF

$+8-2t^2$

$$a(t) = \frac{-2t(t^2+4)[t^2+4+2(4-t^2)]}{(t^2+4)^4}$$

$$a(t) = \frac{-2t\cancel{(t^2+4)}(12-t^2)}{(t^2+4)^{\cancel{4}\,3}} = \frac{-2t(12-t^2)}{(t^2+4)^3}$$

(b) $s(1) = \frac{1}{1^2+4} = \frac{1}{5}$ ft

$v(1) = \frac{4-(1)^2}{(1^2+4)^2} = \frac{3}{25}$ ft/s

speed(1) = $3/25$ ft/s

$a(1) = \frac{-2(1)(12-1^2)}{(1^2+4)^3} = \frac{-22}{125}$ ft/s^2

(c) $v(t) = 0$

$$\frac{4-t^2}{(t^2+4)^2} = 0$$

$$4 - t^2 = 0$$
$$-t^2 = -4$$
$$t^2 = 4$$
$$t = \pm 2 \text{ s}$$

e) $s(0) = 0$

$s(2) = \frac{2}{2^2+4} = \frac{2}{8} = \frac{1}{4} \rightarrow \frac{29}{116}$ $\quad \frac{29}{116}$

$s(5) = \frac{5}{5^2+4} = \frac{5}{29} \rightarrow \frac{20}{116}$ $\quad \frac{9}{116}$

$\frac{38}{116}$ ft

$\frac{19}{58}$ ft