Do Now: From the calculator active section of the 1997 multiple choice

80. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

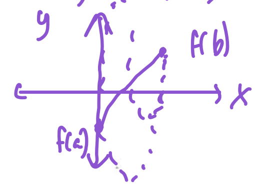
(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

Sample Practice Problems for the Topic of Motion

Intermediate Value Thm (IVT)
A function $y=f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value b/w $f(a)$ and $f(b)$



Example 1 (numerical).

The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.
continuous

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

left b/c $v(0) < 0$

2. Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Explain your answer.

Yes by the IVT. Since $v(0) < 0$ and $v(2) > 0$ there has to be a time where $v(t) = 0$ b/w 0 and 2.

3. Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.

$$v'(10) = \frac{v(12)-v(8)}{12-8} = \frac{5-7}{4} = -\frac{1}{2} \text{ m/min}^2$$

the acceleration at $t = 10$ min

4. Let $a(t)$ denote the acceleration of the particle at time t. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify your answer.

MVT holds so it's going to be true if average acceleration = 0

Yes b/w $[6,12]$

average acc: $\frac{v(12)-v(6)}{12-6} = \frac{5-5}{6} = 0$

Example 2 (Numerical)

Motion Problem

$v(t)$ is differentiable

Time(min)	0	2	4	7	9	10
velocity v(t) meters/min	5	6	8	3	-3	-5

1) Using the table state a value of t when the particle is moving to the left. Justify your choice.
$t = 9$ min b/c $v(9) < 0$ or
$t = 10$ min b/c $v(10) < 0$

2) Is there a time during the interval $0 \le t \le 10$ minutes when the particle is at rest? Explain.
By the IVT since $v(7) > 0$ and $v(9) < 0$ there has to be a time where $v(t) = 0$ in that interval.

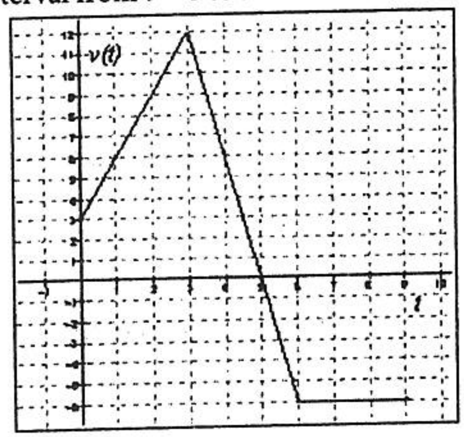
3) Use the table to approximate $v'(3)$, indicating appropriate units. What does $v'(3)$ mean in terms of the motion of the particle?

$$v'(3) \approx \frac{v(4) - v(2)}{4-2} = \frac{8-6}{2} = \frac{2}{2} = 1 \text{ m/min}^2$$

the acceleration of the particle at $t = 3$ min

Example 1 (graphical)

The graph below represents the velocity v, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ to $t = 9$ seconds.



1. At $t = 4$ seconds, is the particle moving to the right or left? Explain your answer.

Right b/c $v(4) > 0$.

2. Over what time interval is the particle moving to the left? Explain your answer.

$(5, 9]$ b/c $v(t)$ is negative in that interval.

3. At $t = 4$ seconds, is the acceleration of the particle positive or negative? Explain your answer.

At $t=4$ the acceleration is negative b/c velocity is decreasing at $t=4$

4. What is the average acceleration of the particle over the interval $2 \le t \le 4$? Show the computations that lead to your answer and indicate units of measure.

$$\frac{v(4)-v(2)}{4-2} = \frac{6-9}{2} = \frac{-3}{2} \text{ ft/s}^2$$

5. Is there guaranteed to be a time t in the interval $2 \le t \le 4$ such that $v'(t) = -3/2$ ft/sec²? Justify your answer.

No the MVT does not apply here b/c this function is not diff. on $(2,4)$. Namely it is not diff. at $t=3$

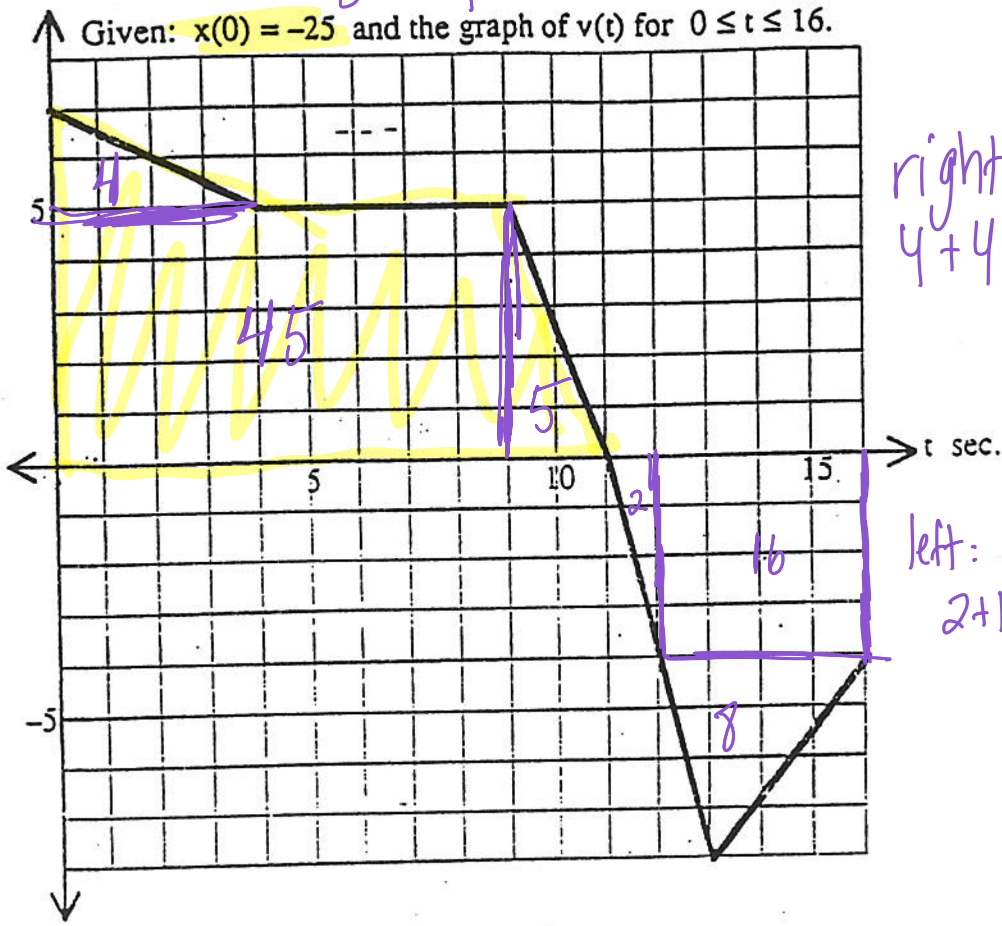
6. At what time t in the given interval is the particle farthest to the right? Explain your answer.

At $t = 5$ seconds b/c the particle moves to the right until $t = 5$ s then it moves left.

Example 2 (graphical)

position at $t=0$ is -25

Given: $x(0) = -25$ and the graph of $v(t)$ for $0 \le t \le 16$.



right:
$4+45+5=54$

left:
$2+16+8=26$

(1) Does the particle begin moving right or left? right b/c $v(0)>0$

(2) When is the particle at rest? $t=11$ b/c $v(11)=0$

(3) What is the maximum velocity? 7 ft/s

(4) What is the maximum speed of the particle? speed = |velocity| $|-8| = 8$ ft/s

(5) When is the particle moving to the left? $(11, 16]$ b/c $v(t)<0$

(6) How far to the right does the particle get? $x(0)$ $-25+54=29$ ft

(7) How far does the particle travel? $54+26=80$ ft

(8) What is the particle's finishing position? $-25+54=29-26=3$ ft

(9) On what interval is the particle speeding up? ______

to be continued...

Homework 01-09 and 01-10

(11) $s(t) = t^3 - 6t^2, \quad t \geq 0$

a) $v(t) = 3t^2 - 12t$

$a(t) = 6t - 12$

b) $s(1) = 1^3 - 6(1)^2 = 1 - 6 = -5$ ft

$v(1) = 3(1)^2 - 12(1) = -9$ ft/s

speed$(1) = 9$ ft/s

$a(1) = 6(1) - 12 = -6$ ft/s^2

c) $v(t) = 0$

$3t^2 - 12t = 0$

$3t(t-4) = 0$

$t = 0s \mid t = 4s$

d) $v(t)$: $-$ on $(0, 4)$, $+$ after 4

$a(t)$: $-$ on $(0, 2)$, $+$ after 2

$a(t) = 0$

$6t - 12 = 0$

$t = 0$

speeding ↑ $(0, 2), (4, \infty)$

slowing ↓ $(2, 4)$

e) $s(0) = 0$

$s(4) = -32$

$s(5) = -25$

$\left.\right\} 32$

$\left.\right\} 7$

39 ft

(12) $s(t) = t^4 - 4t + 2$

a) $v(t) = 4t^3 - 4$

$a(t) = 12t^2$

b) $s(1) = 1 - 4 + 2 = -1$ ft

$v(1) = 4(1)^3 - 4 = 0$ ft/s

speed$(1) = 0$ ft/s

$a(1) = 12$ ft/s^2

c) $4t^3 - 4 = 0$

$4(t^3 - 1) = 0$

$4(t-1)(t^2+t+1) = 0$

$t = 1s \mid t = \frac{-1 \pm \sqrt{1-4}}{2}$

imaginary

d) $v(t)$: $-$ on $(0, 1)$, $+$ after 1

$a(t)$: $+$ after 0

$a(t) = 0$ at $t = 0$

speeding ↑ $(1, \infty)$

slowing ↓ $(0, 1)$

e) $s(0) = 2$

$s(1) = -1$

$s(5) = 607$

$\left.\right\} 3$

$\left.\right\} 608$

611 ft

(13) (e) $s(0) = 3\cos(\frac{\pi}{2} \cdot 0) = 3\cos(0) = 3$

$s(2) = 3\cos(\frac{\pi}{\cancel{2}} \cdot \cancel{2}) = -3$

$s(4) = 3\cos(\frac{\pi}{2} \cdot 4) = 3\cos(2\pi) = 3$

$s(5) = 3\cos(\frac{\pi}{2} \cdot 5) = 3\cos(\frac{5\pi}{2}) = 0$

$\left.\begin{matrix} 3 \\ -3 \end{matrix}\right\} 6 \quad \left.\begin{matrix} -3 \\ 3 \end{matrix}\right\} 6 \quad \left.\begin{matrix} 3 \\ 0 \end{matrix}\right\} 3$

15 ft

(14) $s(t) = \frac{t}{t^2+4}$

(a) $v(t) = \frac{(t^2+4)(1) - \overbrace{t(2t)}^{-2t^2}}{(t^2+4)^2} = \frac{4-t^2}{(t^2+4)^2}$

$a(t) = \frac{(t^2+4)^2(-2t) - (4-t^2) \cdot 2(t^2+4) \cdot 2t}{((t^2+4)^2)^2}$

$a(t) = \frac{-2t(t^2+4)^2 - 4t(t^2+4)(4-t^2)}{(t^2+4)^4}$ * factor out GCF

$a(t) = \frac{-2t(t^2+4)[t^2+4+\overbrace{2(4-t^2)}^{+8-2t^2}]}{(t^2+4)^4}$

$a(t) = \frac{-2t\cancel{(t^2+4)}(12-t^2)}{(t^2+4)^{\cancel{4}\,3}} = \frac{-2t(12-t^2)}{(t^2+4)^3}$

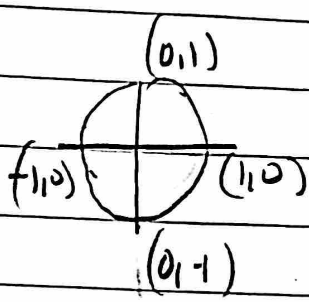
(b) $s(1) = \frac{1}{1^2+4} = \frac{1}{5}$ ft

$v(1) = \frac{4-(1)^2}{(1^2+4)^2} = \frac{3}{25}$ ft/s

speed(1) $= 3/25$ ft/s

$a(1) = \frac{-2(1)(12-1^2)}{(1^2+4)^3} = \frac{-22}{125}$ ft/s^2

(13) $s(t) = 3\cos\left(\frac{\pi}{2}t\right)$



a) $v(t) = -3\sin\left(\frac{\pi}{2}t\right)\cdot\frac{\pi}{2} = -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}t\right)$

$a(t) = -\frac{3\pi}{2}\cos\left(\frac{\pi}{2}t\right)\cdot\frac{\pi}{2} = \frac{-3\pi^2}{4}\cos\left(\frac{\pi}{2}t\right)$

b) $s(1) = 3\cos\left(\frac{\pi}{2}(1)\right) = 3\cos\left(\frac{\pi}{2}\right) = 3(0) = 0$ ft

$v(1) = -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}(1)\right) = -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}\right) = -3\pi/2$ ft/s

speed(1) $= 3\pi/2$ ft/s

$a(1) = \frac{-3\pi^2}{4}\cos\left(\frac{\pi}{2}(1)\right) = \frac{-3\pi^2}{4}\cos\left(\frac{\pi}{2}\right) = \frac{-3\pi^2}{4}(0) = 0$ ft/s^2

c) $v(t) = 0$

$\frac{2}{3\pi}\cdot -\frac{3\pi}{2}\sin\left(\frac{\pi}{2}t\right) = 0\cdot\frac{-2}{3\pi}$

$\sin\left(\frac{\pi}{2}t\right) = 0$

$\frac{\pi}{2}t = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$

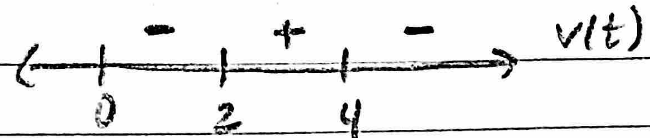
$\frac{\pi}{2}t = 0$ $\quad$ $\frac{\pi}{2}t = \pi$ $\quad$ $\frac{\pi}{2}t = 2\pi$ $\quad$ $\frac{\pi}{2}t = 3\pi$

$t = 0$ $\quad$ $\frac{1}{2}t = 1$ $\quad$ $\frac{1}{2}t = 2$ $\quad$ $\frac{1}{2}t = 3$

$t = 2$ $\quad$ $t = 4$ $\quad$ $\cancel{t = 6}$ outside dof $0 \le t \le 5$

$t = 0, 2, 4$ sec

d)



speeding ↑ (0,1), (2,3), (4,5)

slowing ↓ (1,2), (3,4)

$a(t) = 0$

$\frac{-3\pi^2}{4}\cos\left(\frac{\pi}{2}t\right) = 0$

$\cos\left(\frac{\pi}{2}t\right) = 0$

$\frac{\pi}{2}t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$\frac{\pi}{2}t = \frac{\pi}{2}$ $\quad$ $\frac{\pi}{2}t = \frac{3\pi}{2}$ $\quad$ $\frac{\pi}{2}t = \frac{5\pi}{2}$ $\quad$ $\frac{\pi}{2}t = \frac{7\pi}{2}$

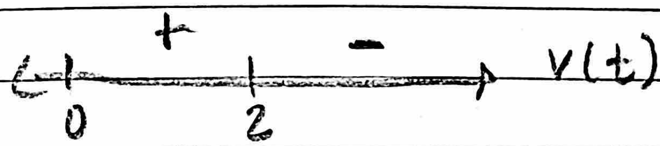
$t = 1$ $\quad$ $t = 3$ $\quad$ $t = 5$ $\quad$ $\cancel{t = 7}$ outside domain

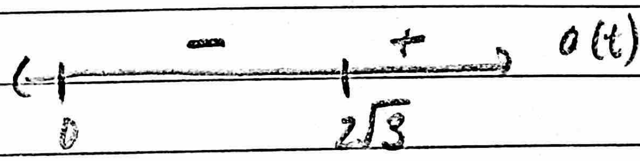
(c) $v(t) = 0$

$$\frac{4-t^2}{(t^2+4)^2} = 0$$

$$4 - t^2 = 0$$
$$-t^2 = -4$$
$$t^2 = 4$$
$$t = \pm 2 \text{ s}$$

(d)





speeding ↑: $(2, 2\sqrt{3})$
slowing ↓: $(0,2), (2\sqrt{3}, \infty)$

$a(t) = 0$

$$\frac{-2t(12-t^2)}{(t^2+4)^3} = 0$$

$$-2t(12-t^2) = 0$$

$t = 0$ | $12 = t^2$

$\pm 2\sqrt{3} = t$

e) $s(0) = 0$

$s(2) = \frac{2}{2^2+4} = \frac{2}{8} = \frac{1}{4} \rightarrow \frac{29}{116}$ $\quad \frac{29}{116}$

$s(5) = \frac{5}{5^2+4} = \frac{5}{29} \rightarrow \frac{20}{116}$ $\quad \frac{9}{116}$

$\frac{38}{116}$ ft

$\frac{19}{58}$ ft