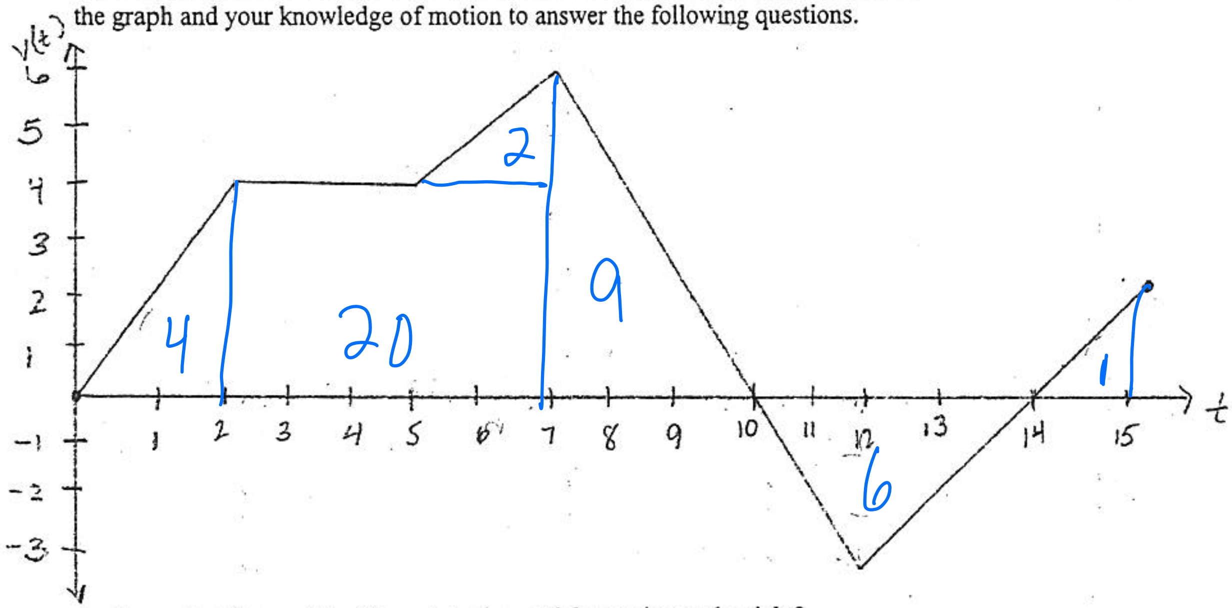
Do Now: 1,2, 4 and 5

Example 3 (graphical)

The graph shown below shows the velocity, v (t), of a particle moving along the x-axis for $0 \le t \le 15$. Use the graph and your knowledge of motion to answer the following questions.



1. Over what interval(s) of time, t, is the particle moving to the right?

(0,10) and (14, 15] b/c v(t) > 0

2. At which time(s) is the particle at rest?

t = 0, 10, 14 b/c v(t) = 0 at those pts.

3. Over what time interval(s) is the speed decreasing? Explain your answer.

(7,10) and (12,14) b/c v(t) and a(t) have diff signs over those intervals.

4. Find the total distance traveled by the particle over the time interval $0 \le t \le 15$.

4 + 20 + 2 + 9 + 6 + 1 = 42 units

5. Find the total displacement of the particle over the time interval $0 \le t \le 15$.

35 − 6 + 1

35 units right then 6 units left 1 right = 30

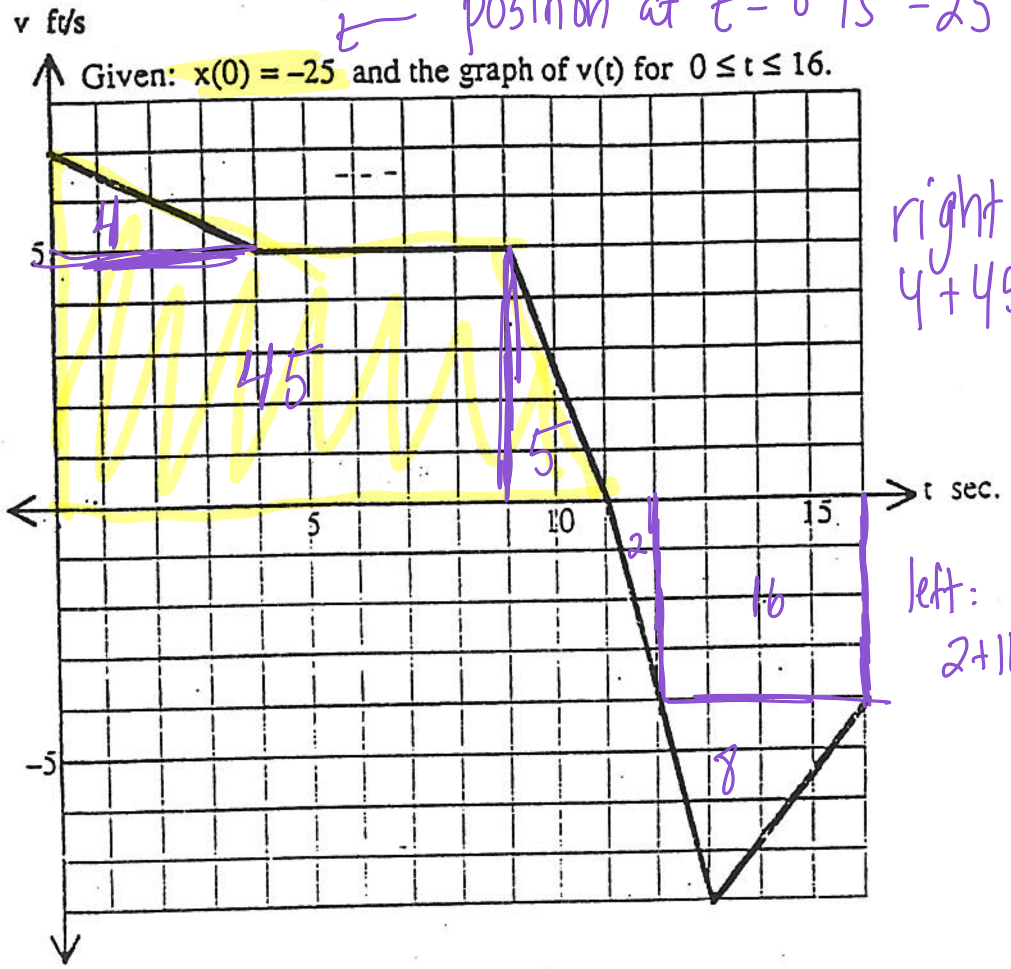
6. If the initial position of the particle is x (0) = 5, find the position of the particle at time t = 15.

x(15) = 5 + 30 = 35

Example 2 (graphical)

← position at $t=0$ is -25

Given: $x(0) = -25$ and the graph of $v(t)$ for $0 \le t \le 16$.



right: $4+45+5=54$

left: $2+16+8=26$

(1) Does the particle begin moving right or left? right b/c $v(0)>0$

(2) When is the particle at rest? $t=11$ b/c $v(11)=0$

(3) What is the maximum velocity? 7 ft/s

speed = |velocity|

(4) What is the maximum speed of the particle? $|-8| = 8$ ft/s

(5) When is the particle moving to the left? $(11, 16]$ b/c $v(t)<0$

x(0) ←

(6) How far to the right does the particle get? $-25+54=29$ ft

(7) How far does the particle travel? $54+26=80$ ft

(8) What is the particle's finishing position? $-25+54=29-26=3$ ft

(9) On what interval is the particle speeding up? $(11, 13]$

to be continued...

Do Now

① The position of a moving particle is $s(t) = \frac{t^3}{6} - \frac{t^2}{2} + t - 3$, $t \geq 0$. Find the the maximum speed of the particle on the interval $[0,4]$

We need min. & max vel.

$v(t) = \frac{1}{2}t^2 - t + 1$

$v'(t) = t - 1$

$t - 1 = 0$

$t = 1$

$v(0) = 1$

$v(1) = \frac{1}{2} - 1 + 1 = \frac{1}{2}$

$v(4) = 8 - 4 + 1 = 5$

max speed $= 5$

② Find the value of c that satisfies the MVT on the interval $[0,5]$ for the function $f(x) = x^3 - 6x$.

cont $[0,5]$ ✓

diff $(0,5)$ ✓

$f'(x) = 3x^2 - 6$

$3c^2 - 6 = \frac{f(5) - f(0)}{5 - 0}$

$c = \frac{5}{\sqrt{3}}$

$3c^2 - 6 = \frac{95 - 0}{5}$

$3c^2 - 6 = 19$

$3c^2 = 25$

$c^2 = \frac{25}{3}$

$c = \cancel{\pm}\frac{5}{\sqrt{3}}$

③ A particle moves along a horizontal line so that at any time $t \geq 0$, its position is given by $x(t) = (t+1)(t-3)^3$, where t is in time in seconds and $x(t)$ is in feet. At time $t = 4$, is the particle speeding up or slowing down?

$$v(t) = 3(t+1)(t-3)^2 + (t-3)^3 = (t-3)^2(3t+3+t-3)$$

$$(t-3)^2(4t)$$

$v(4) > 0$

$$a(t) = 4(t-3)^2 + 4t \cdot 2(t-3)$$

$a(4) > 0$

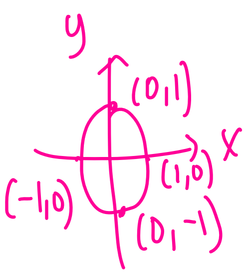
Speeding up b/c $v(4)$ and $a(4)$ are both positive.

④ If $g(x) = \cos\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\pi < x < 3\pi$ that satisfies the conclusion of the MVT. Find c.

$$g'(x) = -\frac{1}{2}\sin\left(\frac{1}{2}x\right)$$

cont $[\pi, 3\pi]$ ✓

diff $(\pi, 3\pi)$ ✓



$$-\frac{1}{2}\sin\left(\frac{1}{2}c\right) = \frac{\overset{0}{g(3\pi)} - \overset{0}{g(\pi)}}{2\pi}$$

$$-\frac{1}{2}\sin\left(\frac{1}{2}c\right) = 0$$

$$\sin\left(\frac{1}{2}c\right) = 0$$

$$\sin A = 0$$

$$A = 0, \pi, 2\pi, 3\pi, \ldots$$

$\frac{1}{2}c = 0$ $\quad$ $\frac{1}{2}c = \pi$ $\quad$ $\frac{1}{2}c = 2\pi$

$\cancel{c = 0}$ $\quad$ $c = 2\pi$ $\quad$ $\cancel{c = 4\pi}$

$$c = 2\pi$$

$a(t) = 12t - 42$

$a(t) = 0$

$t = 7/2$

e) $-$ $+$ $a(t)$; 0, $\frac{7}{2}$

$(\frac{7}{2}, \infty)$

f) $x(1) = -9$ } 11

$x(2) = 2$ } 27

$x(5) = -25$ } 11

$x(6) = -14$

TD: 49 ft

5. The position of a particle moving along the line $y = 2$ is given by $x(t) = 2t^3 - 21t^2 + 60t - 50$ where t is the time in seconds, $t \geq 0$ and x is the position in feet from the point $(0,2)$.

(a) At what time(s) is the particle at rest?
(b) At what time(s) is the particle moving to the right?
(c) At what time(s) is the particle moving to the left?
(d) What is the maximum speed of the particle on the interval $[1,6]$?
(e) On what interval is the velocity of the particle increasing?
(f) What is the total distance travelled by the particle on the interval $[1,6]$?
(g) On what interval(s) is the speed increasing?
(h) On what interval(s) is the speed decreasing?

g) $(5,\infty) \cup (2, 7/2)$

h) $(0,2) \cup (7/2, 5)$

(a) $v(t) = 0$

$6t^2 - 42t + 60 = 0$

$t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$

$t = 5, 2$ s

b) $+$ $-$ $+$ $v(t)$; 0, 2, 5

right: $[0,2) \cup (5,\infty)$

c) left: $(2,5)$

d) $v'(t) = 12t - 42$

$12t - 42 = 0$

$t = 7/2$

$v(1) = 24$

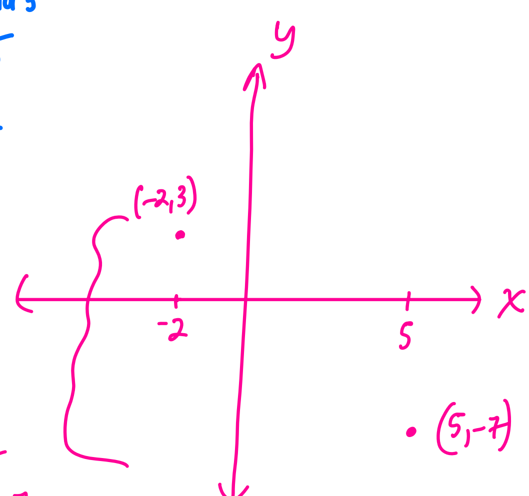
$v(7/2) = -13.5$

$v(6) = 24$

max speed: 24 ft/s

6. Let f be a continuous function on the closed interval $[-2,5]$. $f(-2) = 3$ and $f(5) = -7$, then the IVT guarantees that

(A) $-7 \leq f(x) \leq 3$ for all x between -2 and 5.
(B) $f(c) = -\frac{10}{7}$ for at least one c between -2 and 5
(C) $f(c) = -3$ for at least one c between -2 and 5
(D) $f(c) = 0$ for at least one c between -7 and 3
(E) $f(x)$ is continually decreasing between -2 and 5



has to hit every y value b/c -7 and 3 in that interval $[-2,5]$

⑦ The approximate value of $y=\sqrt{4+\sin x}$ at $x=.12$ obtained from the tangent line to the graph at $x=0$ is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

$$y(0)=\sqrt{4+\sin 0}=\sqrt{4+0}=\sqrt{4}=2$$

$$y'(x)=\frac{1}{2}(4+\sin x)^{-\frac{1}{2}}\cdot\cos x$$

$$y'(0)=\frac{1}{2}(4+\sin 0)^{-\frac{1}{2}}\cdot\cos 0$$

$$\frac{1}{2}(4)^{-\frac{1}{2}}\cdot 1=\frac{1}{2}\left(\frac{1}{2}\right)\cdot 1=\frac{1}{4}$$

$$y-2=\frac{1}{4}(x-0)$$

$$y-2=\frac{1}{4}x$$

at $x=.12$

$$y-2=\cancel{\frac{1}{4}}(\overset{.03}{\cancel{.12}})$$

$$y-2=.03$$

$$y=2.03$$

Homework 01-11

1990 BC-1

$x(t) = (t-1)^3(2t-3)$

(a) $v(t) = 3(t-1)^2 \cdot (2t-3) + (t-1)^3 \cdot 2$

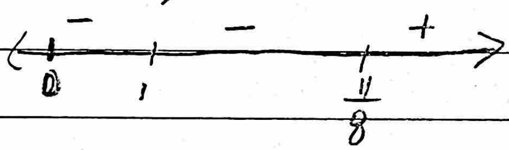
$v(t) = 3(t-1)^2(2t-3) + 2(t-1)^3$

(b) $v(t) = (t-1)^2\left(3(2t-3) + 2(t-1)\right)$

$(t-1)^2(6t-9+2t-2)$

$v(t) = (t-1)^2(8t-11)$

$(t-1)^2(8t-11) < 0$



$(0,1),\ (1, \frac{11}{8})$

when $0 < t < 11/8$, $v(t) < 0$ $t \neq 1$

(c) $a(t) = v'(t) = (t-1)^2 \cdot 8 + 2(t-1)(8t-11)$

$0 = 8(t-1)^2 + 2(t-1)(8t-11)$

$0 = (t-1)\left(8(t-1) + 2(8t-11)\right)$

$0 = (t-1)(8t-8+16t-22)$

$0 = (t-1)(24t-30)$

$t = 1$ | $t = 30/24 = 5/4$

$V(1) = 3(\overset{0}{1-1})^2 \cdot (2(1)-3) + 2(\overset{0}{1-1})^3 = 0$ particle is not moving since $v(t) = 0$

$V(5/4) = 3(5/4-1)^2 \cdot (2(5/4)-3) + 2(5/4-1)^3$

$= 3(\frac{1}{4})^2 \cdot (10/4 - 3) + 2(\frac{1}{4})^3$

$3(1/16) \cdot (-2/4) + 2(\frac{1}{64})$

$\frac{3}{16} \cdot (-\frac{1}{2}) + \frac{1}{32} = -\frac{3}{32} + \frac{1}{32} = \frac{-2}{32} = \frac{-1}{16}$

$\therefore$ when $t = 5/4$ $v(t) \neq 0$ so particle is moving.

1993 AB-2

(a) $x(t) = 2te^{-t}$

$v(t) = 2te^{-t} \cdot -1 + 2e^{-t} = -2te^{-t} + 2e^{-t} = 2e^{-t}(-t+1)$

$a(t) = 2e^{-t} \cdot -1 + (-t+1)2e^{-t} \cdot -1 \qquad = -2e^{-t}(1 + (-t+1))$ $(-t+2)$

$-2e^{-t} - 2e^{-t}(-t+1)$

$-2e^{-t} + 2te^{-t} - 2e^{-t} = -4e^{-t} + 2te^{-t}$

$a(0) = -4e^{0} + 2(0)e^{0} = -4$

(b) $-4e^{-t} + 2te^{-t} = 0$

$e^{-t}(2t-4) = 0$

$e^{-t} \neq 0 \quad | \quad t = 2$

$V(2) = 2e^{-2}(-2+1) = -2e^{-2} = -2/e^{2}$

(c) $v(t) = 0$

$2e^{-t}(-t+1) = 0$

$2e^{-t} \neq 0 \quad | \quad t = 1$

$x(0) = 2(0)e^{0} = 0$

$x(1) = 2(1)e^{-1} = 2/e$

$x(5) = 2(5)e^{-5} = 10/e^{5}$

$\frac{2}{e}$

$|x(1) - x(0)|$

$|2/e - 0| = 2/e$

$|x(5) - x(1)|$

$\left|\frac{10}{e^5} - \frac{2}{e}\right|$ is negative

$\left|\frac{10}{e^5} - \frac{2e^4}{e^5}\right| = \left|\frac{10 - 2e^4}{e^5}\right| = \frac{2e^4 - 10}{e^5}$

$\frac{2}{e} + \frac{2e^4 - 10}{e^5} = \frac{2e^4 + 2e^4 - 10}{e^5} = \boxed{\frac{4e^4 - 10}{e^5}}$