Name: ______________________________________ Date: ____________

AP Calculus AB: Finding Antiderivatives

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Do Now:

1. Given $f'(x) = 2x$, find $f(x)$.

$$f(x) = x^2$$

$$f(x) = x^2 + 1$$

$$\vdots$$

$$\left\{ \begin{array}{l} f'(x) = 2x \\ f(x) = x^2 + c \end{array} \right.$$

The opposite of a derivative is called an antiderivative or integral. If $f(x)$ is the antiderivative of $g(x)$, then

* If there are no upper or lower limits, we call it the general antiderivative

upper limit

$$\int \underbrace{g(x)}_{\text{that function is the integrand}} \boxed{dx} = f(x) + c \xrightarrow{} \text{variable of integration}$$

lower limit

The process of creating such an expression is called antidifferentiation or integration. Why do we have to use a constant of integration?

Because of what we discussed in the do now. There are infinite functions that have the same derivative.

Rules of Integration:

① Power Rule $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

② $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx + c$

③ $\int kf(x)dx = k\int f(x)dx + c$

④ $\int kdx = kx + c$

# Practice

1. $\int(3x+x^2)dx$

$\int 3x\,dx + \int x^2 dx$

$3\int x\,dx + \int x^2 dx$

$3\cdot\frac{x^2}{2} + \frac{x^3}{3} \quad + C$

$\frac{3x^2}{2} + \frac{x^3}{3} + C$

2. $\int(3x^2+4x)dx$

$\cancel{3}\cdot\frac{x^3}{\cancel{3}} + 4\cdot\frac{x^2}{2} + C$

$x^3 + 2x^2 + C$

3. $\int(7x^3+6x^5)dx$

$\frac{7x^4}{4} + \frac{6x^6}{6} + C$

$\frac{7x^4}{4} + x^6 + C$

4. $\int 8dx$

$8x + C$

5. $\int\left(2x^4+\frac{x^3}{3}+\sqrt{x}\right)dx$

$\to \frac{1}{3}x^3 \quad \to x^{\frac{1}{2}}$

$\frac{2x^5}{5}+\frac{1}{3}\cdot\frac{x^4}{4}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C$

$\frac{2x^5}{5}+\frac{x^4}{12}+\frac{2}{3}x^{\frac{3}{2}}+C$

Find the most general antiderivative, $F(x)$, of the function.

6. $f(x)=x-3$

$F(x)=\frac{1}{2}x^2-3x+C$

7. $f(x)=\frac{1}{2}+\frac{3}{4}x^2-\frac{4}{5}x^3$

$F(x)=\frac{1}{2}x+\frac{1}{4}x^3-\frac{1}{5}x^4+C$

8. $f(x) = (x+1)(2x-1)$

$$f(x) = 2x^2 + x - 1$$

$$F(x) = \frac{2x^3}{3} + \frac{x^2}{2} - x + C$$

9. $f(x) = 5x^{\frac{1}{4}} - 7x^{\frac{3}{4}}$

$$F(x) = \cancel{5} \cdot \frac{4}{\cancel{5}} x^{\frac{5}{4}} - \cancel{7} \cdot \frac{4}{\cancel{7}} x^{\frac{7}{4}} + C$$

$$F(x) = 4x^{\frac{5}{4}} - 4x^{\frac{7}{4}} + C$$

10. $f(x) = 6\sqrt{x} - \sqrt[6]{x}$

$$= 6x^{\frac{1}{2}} - x^{\frac{1}{6}}$$

$$F(x) = \overset{2}{\cancel{6}} \cdot \frac{2}{\cancel{3}} x^{\frac{3}{2}} - \frac{6}{7} x^{\frac{7}{6}} + C$$

$$= 4x^{3/2} - \frac{6}{7} x^{\frac{7}{6}} + C$$

$$= 4x\sqrt{x} - \frac{6}{7} x\sqrt[6]{x} + C$$

$$x^{3/2} = \sqrt{x^3}$$

$$x\sqrt{x}$$

$$x^{7/6} = \sqrt[6]{x^7}$$

$$= x\sqrt[6]{x}$$

11. $f(x) = \frac{10}{x^9}$

$$f(x) = 10x^{-9}$$

$$F(x) = 10\,\frac{x^{-8}}{-8} = -\frac{5}{4} x^{-8} + C$$

$$= -\frac{5}{4x^8} + C$$

# Homework 01-14

## Linear Motion Review Answer Key

① $x(t) = 2t^3 - 21t^2 + 60t - 50$

(a) $v(t) = 0$

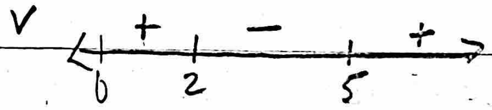
$6t^2 - 42t + 60 = 0$

$6(t^2 - 7t + 10) = 0$

$6(t-5)(t-2) = 0$

$t = 5 \mid t = 2$ The particle is at rest at $t = 2$ and at $t = 5$ sec.

(b) $v(t) > 0$



The particle is moving to the right at $0 < t < 2$ or $t > 5$

(c) The particle is moving to the left on $2 < t < 5$

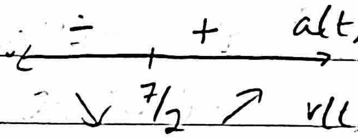
(d) Max min on closed interval. Find critical pts.

$v'(t) = 2t - 7$ (use $t^2 - 7t + 10 = 0$)

$0 = 2t - 7$

$t = 7/2$

(e)



The particle is increasing if $t > 7/2$.

candidate test/closed interval test:

| $t$ | 1 | 7/2 | 6 |
|---|---|---|---|
| $v(t)$ | 24 | -13.5 | 24 |
| speed $\lvert v(t)\rvert$ | 24 | 13.5 | 24 |

∴ maximum speed is 24 ft/sec.

(f) $v(t) = 0$ at $t = 2$ and $t = 5$

$s(1) = -9$

$s(2) = 2$

$s(5) = -25$

$s(6) = -14$

T.D. $= |s(2) - s(1)| + |s(5) - s(2)| + |s(6) - s(5)|$

$\boxed{T.D. = 11 + 27 + 11 = 49 \text{ ft}}$

(g) $2t^3 - 21t^2 + 60t + 50 = 10$

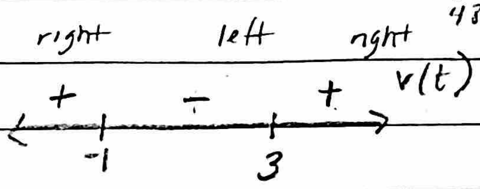
$2t^3 - 21t^2 + 60t + 40 = 0$

Use 2ND calc. to solve $t = 6.682$ sec.

② $x(t) = t^3 - 3t^2 - 9t + 2$

$3(4)^2 - 6(4) - 9$
$48 - 24 - 9$

(a) $v(t) = 3t^2 - 6t - 9$
$0 = 3(t^2 - 2t - 3)$
$0 = 3(t+1)(t-3)$
$t = -1 \mid t = 3$



So at $t = 3$ the particle changes direction b/c its velocity changed sign.

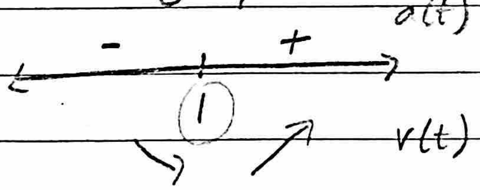
(b) $x(0) = 2$
$x(3) = 27 - 3(3)^2 - 9(3) + 2 = 27 - 27 - 27 + 2 = -25$
$x(4) = 64 - 3(4)^2 - 9(4) + 2 = 64 - 48 - 36 + 2 = -18$

$T.D. = |x(3) - x(0)| + |x(4) - x(3)| = |-25 - 2| + |-18 - (-25)| = 27 + 7 = 34$

(c) $v'(t) = a(t) = 6t - 6$
$a(2) = 6(t-1) = 6(2-1) = 6$ ft/sec$^2$

(d) Speed is a max when velocity is a max or min. Need to find c.pts.

$a(t) = 6t - 6 = 0$
$t = 1$



$v(1) = -12$
$v(-1) = 0$
$v(3) = 0$

max speed of 12 ft/sec at $t = 1$.