$$f'(c) = \lim_{x \to 0} \frac{f(c+h) - f(c)}{h}$$

Midterm Review Packet Key

1. (B) (for (D) pt of inflection would make $f''(x)=0$.) 2. (A) 3. (B)

4. $f'(1)$ for $f(x) = x^6$, $f'(x) = 6x^5$, $f'(1) = 6$ (C)

5. $f'(8)$ for $f(x) = x^{1/3}$, $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$, $f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$ (B)

6. $f'(e)$ for $f(x) = \ln x$, $f'(x) = \frac{1}{x}$, $f'(e) = \frac{1}{e}$ (B)

7. $\lim_{x \to 0} \frac{\cos(0+x) - \cos 0}{x} = f'(0)$ for $f(x) = \cos x$, $f'(x) = -\sin x$, $f'(0) = 0$ (B)

8.

(a)

$f'(x)$: + on (0,1), + on (1,2), − on (2,3), − on (3,4)

$f(x)$: ↗ ↗ ↘ ↘

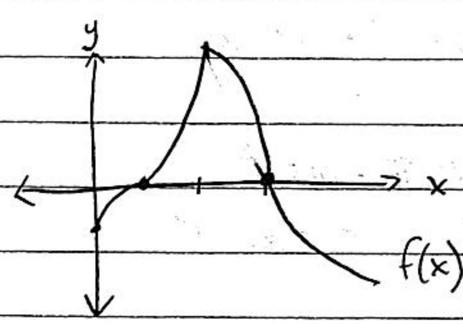
f has a relative maximum at $x=2$ b/c f' changes from ⊕ to ⊖ at $x=2$

(b)

f'': − on (0,1), + on (1,2), − on (2,3), + on (3,4)

$f(x)$: ∩ ∪ ∩ ∪

pt of infl. pt of in pt of inf.



9. $y^2 = 2 + xy$

(a) $2y \, dy/dx = x \, dy/dx + y$

$dy/dx(2y - x) = y$

$dy/dx = \frac{y}{2y - x}$

(b) $\frac{y}{2y-x} = \frac{1}{2}$

$2y = 2y - x$

$0 = -x$

$x = 0$

If $x = 0$, $y^2 = 2$, $y = \pm\sqrt{2}$

so $(0, \pm\sqrt{2})$

(c) $\frac{y}{2y-x} \neq 0$

$y = 0$

$0^2 \neq 2 + 0$ so the curve does not have a horizontal tangent.

$y^2 = 2 + xy$

or $\quad 2y\frac{dy}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} \Rightarrow 2(3)(6) = \frac{7}{3}(6)^2 + 3\frac{dx}{dt}$

$36 = 14 + 3\frac{dx}{dt}$

$22 = 3\frac{dx}{dt}$ (2)

when $y = 3$, $3^2 = 2 + 3x$

$7 = 3x$, $x = 7/3$ $\quad \frac{22}{3} = \frac{dx}{dt}$

(d) $dy/dt = dy/dx \cdot dx/dt$

$6 = \frac{y}{2y - x} \cdot \frac{dx}{dt}$

At $t = 5$

$6 = \frac{3}{6 - 7/3} \cdot dx/dt$

$6 = \frac{3}{11/3} \cdot dx/dt$

$6 = 9/11 \; dx/dt$

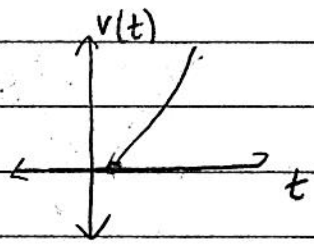
$22/3 = dx/dt$

(10) (D)

(11) (A)

(12) (a) $\frac{x(3) - x(1)}{3 - 1} = \frac{e^3 - \sqrt{3} - (e - 1)}{2} = \frac{e^3 - e - \sqrt{3} + 1}{2} = 8.318$ ft/sec

(b) $v(t) = e^t - \frac{1}{2\sqrt{t}}$, $v(1) = e - \frac{1}{2\sqrt{1}} = e - \frac{1}{2} = 2.218$ ft/sec to the right

(c) 

particle moves to the right when $v(t) > 0$

$t > 0.176$

(d) velocity is zero when $t = 0.17586786\ldots$

$x(0.17586786\ldots) = 0.773$ ft

(13) $x = -.3906462 \approx -0.391$ (C)

(14) $f'(x) = 0$ or dne Three (B)

(15) $f'(x) = 4x^3 + 4x$ $\quad f'(x) = 1$ $\quad f(.237)$

$x \approx .237$ $\quad$ pt $(.237, .115)$

$y - .115 = 1(x - .237)$ $\quad$ (D)

$y = x - .237 + .115 \Rightarrow y = x - .122$

(16) $f(x) = \cos(2x) + \ln(3x)$ $\quad$ NDeriv twice

$f''(x) = 0$ $\quad x \approx .932$ (B)

(17) $f(x) = \sqrt[5]{x^3 - 2x}$

$f'(\sqrt{3}) = .90215$ (B)

18 $f(x)=5x^3+x \qquad g(x)=f^{-1}(x)$, find $g'(6)$

$$5x^3+x=6$$
$$x=1$$

$$f'(x)=15x^2+1$$
$$f'(1)=15(1)^2+1=16$$

$$g'(6)=\frac{1}{f'(1)}=\frac{1}{16}$$