

Name: _____
AP Calc AB: Some Special Antiderivatives

Date: _____
Ms. Loughran

$$f(x) = \ln(-x)$$
$$f'(x) = -\frac{1}{x} \cdot -1 = \frac{1}{x}$$

Do Now:

1. $\int \frac{1}{x} dx = \ln|x| + C$

2. $\int e^x dx = e^x + C$

3. $\int \cos x dx = \sin x + C$

4. $\int \sin x dx = -\cos x + C$

Special Antiderivatives

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

Exercises and Problems

Exercises

In Exercises 1-21, find an antiderivative.

1. $f(x) = 5$

3. $f(x) = x^2$

5. $h(t) = \cos t$

7. $h(z) = \frac{1}{z}$

9. $g(z) = \frac{1}{z^3}$

11. $g(t) = \sin t$

13. $p(t) = t^3 - \frac{t^2}{2} - t$

15. $p(t) = \cos t + \frac{1}{\cos^2 t}$

17. $p(\theta) = 2 \sin(2\theta)$

19. $q(t) = (t+1)^2$

21. $f(x) = 5x - \sqrt{x}$

2. $f(x) = 5x$

4. $g(t) = t^2 + t$

6. $g(z) = \sqrt{z}$

8. $r(t) = \frac{1}{t^2}$

10. $f(z) = e^z$

12. $f(t) = 2t^2 + 3t^3 + 4t^4$

14. $q(y) = y^4 + \frac{1}{y}$

16. $f(t) = \frac{t^2 + 1}{t}$

18. $r(t) = e^t + 5e^{5t}$

20. $f(x) = 5^x$

In Exercises 22-32, find the general antiderivative.

22. $f(t) = 6t$

24. $f(x) = x^2 - 4x + 7$

26. $f(z) = z + e^z$

28. $g(x) = \sin x + \cos x$

30. $p(t) = 2 + \sin t$

32. $g(x) = \frac{5}{x^3}$

23. $h(x) = x^3 - x$

25. $r(t) = t^3 + 5t - 1$

27. $g(t) = \sqrt{t}$

29. $h(x) = 4x^3 - 7$

31. $p(t) = \frac{1}{\sqrt{t}}$

In Exercises 33-40, find an antiderivative $F(x)$ with $F'(x) = f(x)$ and $F(0) = 0$. Is there only one possible solution?

33. $f(x) = 3$

35. $f(x) = -7x$

37. $f(x) = x^2$

39. $f(x) = 2 + 4x + 5x^2$

34. $f(x) = 2x$

36. $f(x) = \frac{1}{4}x$

38. $f(x) = \sqrt{x}$

40. $f(x) = \sin x$

Find the indefinite integrals in Exercises 41-59.

41. $\int 5x \, dx$

43. $\int \sin \theta \, d\theta$

45. $\int \left(t^2 + \frac{1}{t^2}\right) dt$

47. $\int (x^2 + 5x + 8) \, dx$

49. $\int (4t + 7) \, dt$

51. $\int 5e^z \, dz$

53. $\int \sin t \, dt$

55. $\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}}\right) dt$

57. $\int e^{2r} \, dr$

59. $\int \left(\frac{y^2 - 1}{y}\right)^2 dy$

42. $\int x^3 \, dx$

44. $\int (x^3 - 2) \, dx$

46. $\int 4\sqrt{w} \, dw$

48. $\int \frac{4}{t^2} \, dt$

50. $\int \cos \theta \, d\theta$

52. $\int \left(x + \frac{1}{\sqrt{x}}\right) dx$

54. $\int (\pi + x^{11}) \, dx$

56. $\int \cos(x+1) \, dx$

58. $\int \frac{1}{e^z} \, dz$

In Exercises 60-71, evaluate the definite integrals exactly [as in $\ln(3\pi)$], using the Fundamental Theorem, and numerically [$\ln(3\pi) \approx 2.243$]:

60. $\int_0^3 (x^2 + 4x + 3) \, dx$

62. $\int_0^{\pi/4} \sin x \, dx$

64. $\int_2^5 (x^3 - \pi x^2) \, dx$

66. $\int_1^2 \frac{1+y^2}{y} \, dy$

68. $\int_0^{\pi/4} (\sin t + \cos t) \, dt$

70. $\int_{-3}^{-1} \frac{2}{r^3} \, dr$

61. $\int_1^3 \frac{1}{t} \, dt$

63. $\int_0^2 3e^x \, dx$

65. $\int_0^1 \sin \theta \, d\theta$

67. $\int_0^2 \left(\frac{x^3}{3} + 2x\right) dx$

69. $\int_0^1 2e^x \, dx$

71. $\int_0^{\pi/4} \frac{1}{\cos^2 x} \, dx$

Homework 01-14

(7) a) $\int x^8 dx$
 $= \frac{1}{9} x^9 + C$

b) $\int x^{5/7} dx$
 $= \frac{7}{12} x^{12/7} + C$

c) $\int x^3 \sqrt{x} dx$
 $\int x^3 \cdot x^{1/2} dx = \int x^{7/2} dx$
 $= \frac{2}{9} x^{9/2} + C$

9 a) $\int \sqrt[3]{x^2} dx = \int x^{2/3} dx$
 $\frac{3}{5} x^{5/3} + C = \frac{3}{5} x \sqrt[3]{x^2} + C$

b) $\int \frac{1}{x^6} dx = \int x^{-6} dx$
 $= -\frac{1}{5} x^{-5} + C$
 $= -\frac{1}{5x^5} + C$

c) $\int x^{-7/8} dx$
 $= 8 x^{1/8} + C$

9 (a) $\int \frac{1}{2x^3} dx$

(b) $\int (u^3 - 2u + 7) du$

10) $\int (x^{2/3} - 4x^{-1/5} + 4) dx$

$\frac{1}{2} \int x^{-3} dx$

$\int u^3 du - 2 \int u du + \int 7 du$

$\frac{3}{5} x^{5/3} - \frac{4 \cdot 5 x^{4/5}}{4} + 4x + C$

$\frac{1}{2} \frac{x^{-2}}{-2}$

$\frac{1}{4} u^4 - u^2 + 7u + C$

$\frac{3}{5} x \sqrt[3]{x^2} - 5 \sqrt[5]{x^4} + 4x + C$

$-\frac{1}{4} x^{-2} + C = -\frac{1}{4x^2} + C$

11) $\int (x^{-3} + \sqrt{x} - 3x^{1/4} + x^2) dx$

12) $\int (7y^{-3/4} - y^{1/3} + 4y^{1/2}) dy$

$\int x^{-3} dx + \int x^{1/2} dx - 3 \int x^{1/4} dx + \int x^2 dx$

$7 \cdot 4 y^{1/4} - \frac{3}{4} y^{4/3} + 4 \cdot \frac{2}{3} y^{3/2} + C$

$-\frac{1}{2} x^{-2} + \frac{2}{3} x^{3/2} - \frac{12}{5} x^{5/4} + \frac{1}{3} x^3 + C$

$28 y^{1/4} - \frac{3}{4} y^{4/3} + \frac{8}{3} y^{3/2} + C$

14) $\int (2xy^2)^2 dy$

13) $\int x(1+x^3) dx = \int (x+x^4) dx = \frac{1}{2} x^2 + \frac{1}{5} x^5 + C$

$\int (4+4y^2+4y^4) dy$
 $4y + \frac{4}{3} y^3 + \frac{4}{5} y^5 + C$

15) $\int x^{1/3} (2-x)^2 dx = \int (4x^{1/3} - 4x^{4/3} + x^{7/3}) dx = 3x^{4/3} - \frac{12}{7} x^{7/3} + \frac{3}{10} x^{10/3} + C$

17) $\int \frac{x^5 + 2x^2 - 1}{x^4} dx = \int (x + 2x^{-2} - x^{-4}) dx = \frac{1}{2} x^2 - \frac{2}{x} + \frac{1}{3} x^3 + C$

$\frac{x^{-1}}{-1}$
 $\frac{x^{-3}}{-3}$
 $-\frac{1}{x}$

16) $\int (1+x^2)(2-x) dx$

$\int (2+2x^2-x-x^3) dx = 2x + \frac{2}{3} x^3 - \frac{1}{2} x^2 - \frac{1}{4} x^4 + C$

Classwork 01-15

Integration

Exercises + Problems Key

① $F(x) = 5x$

② $F(x) = \frac{5}{2}x^2$

③ $F(x) = \frac{x^3}{3}$

④ $G(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2$

⑤ $H(t) = \sin t$

⑥ $G(z) = \frac{2}{3}\sqrt{z^3}$

⑦ $H(z) = \ln|z|$

⑧ $R(t) = -\frac{1}{t}$

⑨ $G(x) = -\frac{1}{2}x^2$

⑩ $F(z) = e^z$

⑪ $G(t) = -\cos t$

⑫ $F(t) = \frac{2}{3}t^3 + \frac{3}{4}t^4 + \frac{4}{5}t^5$

⑬ $P(t) = \frac{1}{4}t^4 - \frac{1}{6}t^3 - \frac{1}{2}t^2$

⑭ $q(y) = \frac{1}{5}y^5 + \ln|y|$

⑮ $F(t) = \frac{1}{2}t^2 + \ln|t|$

⑰ $q(t) = \frac{1}{3}t^3 + t^2 + t$

⑱ $F(x) = \frac{5}{2}x^2 - \frac{2}{3}x^{3/2}$

⑳ $3t^2 + C$

㉑ $\frac{1}{4}x^4 - \frac{1}{2}x^2 + C$

㉒ $\frac{1}{3}x^3 - 2x^2 + 7x + C$

㉓ $\frac{1}{4}t^4 + \frac{5}{2}t^2 - t + C$

㉔ $\frac{1}{2}z^2 + e^z + C$

㉕ $\frac{2}{3}t^{3/2} + C$

㉖ $-\cos x + \sin x + C$

㉗ $x^4 - 7x + C$

㉘ $2t - \cos t + C$

㉙ $2\sqrt{t} + C$

㉚ $\frac{-5}{2x^2} + C$

41 $\frac{5x^2}{2} + C$

42 $\frac{1}{4}x^4 + C$

43 $-\cos \theta + C$

44 $\frac{1}{4}x^4 - 2x + C$

45 $\frac{1}{3}t^3 - \frac{1}{t} + C$

46 $\frac{2}{3}w^{3/2} + C$

47 $\frac{1}{3}x^3 + \frac{5}{2}x^2 + 8x + C$

48 $\frac{-4}{t} + C$

50 $\sin \theta + C$

52 $\frac{1}{2}x^2 + 2\sqrt{x} + C$

54 $\pi x + \frac{1}{12}x^{12} + C$

55 $\frac{2}{5}t^{5/2} - 2t^{-1/2} + C$

59 $\frac{1}{3}y^3 - 2y + \frac{1}{y} + C$