Name: ____ Date: ___________

AP Calculus AB: More Integration with Trig Functions Ms. Loughran

Do Now:

1. $\int \frac{t+1}{t}\,dt = \int (1+t^{-1})\,dt = t + \ln|t| + C$

2. $\int \cos(3x)\,dx =$

$u = 3x$
$du = 3dx$
$\frac{du}{3} = dx$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin u + C$

$\frac{1}{3}\sin 3x + C$

3. $\int \frac{\sin x}{\cos x}\,dx =$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$-\int u^{-1}\,du$

$-\ln|u| + C$

$-\ln|\cos x| + C$

$\int \tan x\,dx =$

$-\ln|\cos x| + c$

Classwork

4. $\int \frac{\cos x}{\sin x}\,dx =$

$u = \sin x$
$du = \cos x\,dx$

$\int u^{-1}\,du$

$\ln|u| + C$

$\ln|\sin x| + C \rightarrow \int \cot x\,dx$

$\int \tan x\,dx = -\ln|\cos x| + C$ $\int \cot x\,dx = \ln|\sin x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$ $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

5. $\int \frac{\cos x}{\sin^2 x} dx =$

$$\int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u^{-2} du$$

$-u^{-1} + C$

$-\frac{1}{\sin x} + C$

or

$-\csc x + C$

OR $\int \cot x \csc x \, dx$

$-\csc x + C$

6. $\int \sec^3(2x)\tan(2x)dx =$

$$\int \sec^2(2x) \cdot \sec(2x)\tan(2x)\, dx$$

$u = \sec 2x$

$du = 2\sec 2x \tan 2x \, dx$

$$\frac{1}{2}\int u^2 du = \frac{1}{2} \cdot \frac{u^3}{3} + C$$

$$= \frac{1}{6}(\sec(2x))^3 + C$$

$$= \frac{1}{6}\sec^3(2x) + C$$

7. $\int \cot(7x)dx =$ OR $\rightarrow \int \frac{\cos(7x)}{\sin(7x)} dx$

$u = \sin(7x)$

$du = 7\cos(7x)dx$

$\frac{du}{7} = \cos(7x)dx$

$\frac{1}{7}\int u^{-1} du$

$\frac{1}{7}\ln|u| + C$

$\frac{1}{7}\ln|\sin 7x| + C$

$u = 7x$

$du = 7dx$

$\frac{1}{7}\int \cot(u)\, du$

$\frac{1}{7}\ln|\sin u| + C$

$\frac{1}{7}\ln|\sin 7x| + C$

8. $\int \frac{dx}{\cos^2(2x)} =$

$u = 2x$

$du = 2\,dx$

$\int \sec^2 2x \, dx$

$\frac{1}{2}\int \sec^2 u \, du$

$\frac{1}{2}\tan u + C$

$\frac{1}{2}\tan 2x + C$

9. $\int \cot^2(3x)dx =$

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$\int (\csc^2(3x) - 1)\,dx$

$\int \csc^2(3x)\,dx - \int 1\,dx$

$-\frac{1}{3}\cot(3x) - x + C$

10. $\int \cos^3 x dx =$ ← need to rewrite this somehow involving sine

$\int \cos x \cdot \cos^2 x \, dx$

Homework 01-25

(2) 

$V = \pi r^2 h$

$400 = \pi r^2 h$

$\frac{400}{\pi r^2} = h$

$r > 0$

$SA = 2\pi r^2 + 2\pi r h$

$SA(r) = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{400}{\cancel{\pi} r^2}\right)$

$SA(r) = 2\pi r^2 + 800r^{-1}$

$SA'(r) = 4\pi r - 800r^{-2}$

$4\pi r = \frac{800}{r^2}$

$4\pi r^3 = 800$

$r^3 = \frac{800}{4\pi}$

$r = \sqrt[3]{\frac{800}{4\pi}}$

$r = \sqrt[3]{\frac{200}{\pi}}$ cm

$SA''(r) = 4\pi + 1600r^3$

$SA''\left(\sqrt[3]{\frac{200}{\pi}}\right) > 0$

CU

so a

minimum

$r = \sqrt[3]{\frac{200}{\pi}}$ cm

$h = \frac{400}{\pi\left(\sqrt[3]{\frac{200}{\pi}}\right)^2}$ cm

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Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by $k(x+1)^{\frac{1}{2}}$

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

(a)

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} f(x)$ @ $x=3$

$\sqrt{x+1} = 5-x$

$\sqrt{3+1} = 5-3$

$2 = 2$ ✓ cont.

$y = -2x+3$

must be cont @ $x=3$

(c) $k\sqrt{3+1} = m(3)+2$

$2k = 3m+2$

$2(4m) = 3m+2$

$8m = 3m+2$

$5m = 2$

$m = 2/5$

$\frac{1}{2}k(x+1)^{-\frac{1}{2}} \cdot 1 = m$ @ $x=3$

$\frac{k}{2\sqrt{x+1}} = m$

$\frac{k}{2\sqrt{3+1}} = m$

$\frac{k}{4} = m$

$k = 4m$

$k = 4(2/5)$

$k = 8/5$

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The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \ f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

$g'(x) = a e^{ax} + f'(x)$

$g''(0) = a^2 e^{ax} + f''(x)$

$g''(0) = a^2 \cdot e^{a\cdot 0} + f''(0)$

$= a^2 + 3$

(a) $g'(x) = e^{ax} \cdot a + f'(x)$

$g'(0) = e^{a\cdot 0} \cdot a + f'(0)$

$e^0 \cdot a + -4$

$g'(0) = a - 4$

$h(x) = \cos(kx)\, f(x)$

(b) $h'(x) = (-\sin(kx) \cdot k)(f(x)) + \cos(kx) f'(x)$

$h'(x) = -k \sin(k(x))\, f(x) + f'(x) \cos(kx)$

$h'(0) = -k(\sin k(0))\, f(0) + f'(0) \cos 0$

$= 0 + -4 \cdot 1 = -4$

$h'(0) = -4$

$x = 0$

$h(x) = \cos(kx) f(x)$

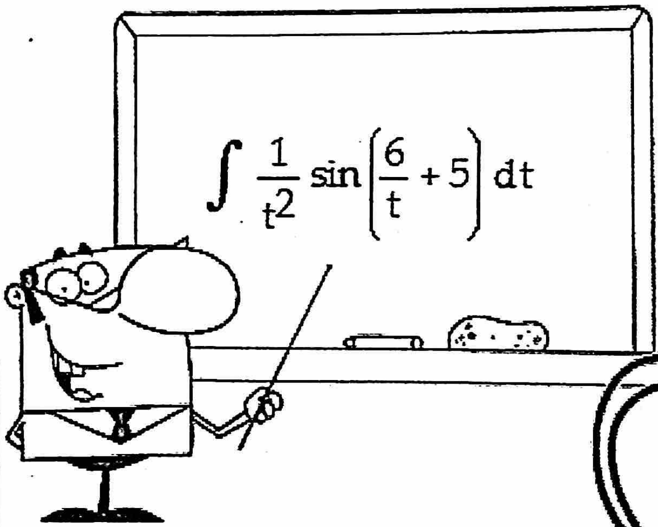
$h(0) = \cos(k \cdot 0) f(0)$

$1 \cdot 2 = 2$

$h(0) = 2$ ✓

$y - 2 = -4(x - 0)$

$y - 2 = -4x$



Answer Key
VERSION 2

1. B
2. C
3. B
4. C
5. C
6. D
7. A
8. C
9. B