Name: _____ Date: _____

AP Calculus AB More Integration Practice Ms. Loughran

Do Now: #s 1 and 2

1. $\int (e^{6\ln x} + e^{\frac{x}{4}})dx =$

$\int (e^{\ln x^6} + e^{\frac{x}{4}})dx$

$\int x^6 + e^{\frac{x}{4}} dx$

$\int x^6 dx + \int e^{\frac{x}{4}} dx \quad u = \frac{1}{4}x$

$\frac{x^7}{7} + 4\int e^u du \quad du = \frac{1}{4}dx$

$\frac{x^7}{7} + 4e^u + C = \frac{x^7}{7} + 4e^{\frac{1}{4}x} + C \quad 4du = dx$

2. $\int x\sqrt{5x^2 - 4}\,dx =$

$u = 5x^2 - 4$

$du = 10x\,dx$

$\frac{du}{10} = x\,dx$

$\frac{1}{10}\int u^{\frac{1}{2}} du$

$\frac{1}{10} \cdot \frac{2}{3}u^{\frac{3}{2}} + C$

$\frac{u^{\frac{3}{2}}}{15} + C$

$\frac{(5x^2 - 4)^{\frac{3}{2}}}{15} + C$

$[\tan^{-1} x] = \frac{1}{1+x^2}$

3. $\int \frac{dx}{9 + x^2} =$

$\int \frac{dx}{9(1 + \frac{x^2}{9})}$

$\frac{1}{9}\int \frac{dx}{1 + \frac{x^2}{9}} = \frac{1}{9}\int \frac{dx}{1 + (\frac{x}{3})^2}$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$3du = dx$

$3 \cdot \frac{1}{9}\int \frac{1}{1+u^2} du$

$\frac{1}{3}\tan^{-1} u + C$

$\frac{1}{3}\tan^{-1}\frac{x}{3} + C$

4. $\int x\sqrt{3x}\,dx =$

$\sqrt{3}\int x^{3/2} dx$

$\sqrt{3} \cdot \frac{2}{5}x^{\frac{5}{2}} + C = \frac{2\sqrt{3}}{5}x^{5/2} + C$

From yesterday...

10. $\int \cos^3 x dx =$ ← need to rewrite this somehow involving sine

$\int \cos x \cdot \cos^2 x\,dx$

$\int \cos x(1 - \sin^2 x)\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int 1 - u^2 du$

$u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$

Homework 01-27

More Practice

1. $\int \cos(8x)\,dx =$

$u = 8x$

$du = 8\,dx$

$\frac{du}{8} = dx$

$\frac{1}{8}\int \cos u\,du = \frac{1}{8}\sin u + C$

$\frac{1}{8}\sin(8x) + C$.

2. $\int \frac{5}{1+x^2}\,dx =$

$5\int \frac{1}{1+x^2}\,dx$

$5\tan^{-1}x + C$

3. $\int \frac{1}{\sqrt{1-x^2}}\,dx =$

$\sin^{-1}(x) + C$

on more at practice sheet # 10

4. $\int \sqrt{x-1}\sqrt{x+1}\,dx =$ $x\,dx$

$u = x^2 - 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \sqrt{x^2-1}\,x\,dx$

$\frac{1}{2}\int u^{\frac{1}{2}}\,du$

$\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$\frac{1}{3}(x^2-1)^{3/2} + C$.

#6 on MI practice BC sheet

5. $\int \sqrt{3+x^2}\,x^3\,dx =$ $x^2 \to x\,dx$

$u = 3 + x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int u^{\frac{1}{2}}\cdot(u-3)\,du$

$\frac{1}{2}\int (u^{3/2} - 3u^{\frac{1}{2}})\,du$

$\frac{1}{2}\left(\frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C\right)$

$\frac{1}{5}u^{5/2} - u^{3/2} + C = \frac{1}{5}(3+x^2)^{5/2} - (3+x^2)^{3/2} + C$.

6. $\int (e^{2x}+1)e^{-x}\,dx =$

$\int (e^x + e^{-x})\,dx$

$\int e^x\,dx + \int e^{-x}\,dx$

$e^x - e^{-x} + C$.

$\int e^u\,du = e^u + C$

$u = -x$

$du = -dx$

$-du = dx$

7. $\int x^2\sec^2(x^3)\,dx =$

$u = x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\frac{1}{3}\int \sec^2 u\,du$

$\frac{1}{3}\tan u + C = \frac{1}{3}\tan(x^3) + C$.

8. $\int \cos(4\theta)\sqrt{2-\sin(4\theta)}\,d\theta =$

$u = 2 - \sin(4\theta)$

$du = -4\cos(4\theta)$

$\frac{-du}{4} = \cos(4\theta)$

$-\frac{1}{4}\int u^{\frac{1}{2}}\,du$

$-\frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{6}(2-\sin 4\theta)^{3/2} + C$

8. $\int \frac{dx}{\cos^2(2x)} =$

$\int \sec^2(2x)\,dx$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$\frac{1}{2}\int \sec^2 u\,du$

$\frac{1}{2}\tan u + C$

$\frac{1}{2}\tan(2x) + C$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

9. $\int \cot^2(3x)dx =$

can't split $\cot(3x)\cdot\cot(3x)$ b/c no product rule we need to get it to something we can integrate

$\int (\csc^2(3x) - 1)\,dx$

$u = 3x$

$du = 3dx$

$\frac{1}{3}\int \csc^2 u\,du - \int 1\,dx$

$-\frac{1}{3}\cot u - x + C = -\frac{1}{3}\cot(3x) - x + C$

10. $\int \cos^3 x dx =$

same idea as 9

$\int \cos^2 x \cdot \cos x)\,dx$

$\int (1 - \sin^2 x)\cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int (1 - u^2)\,du = u - \frac{u^3}{3} + C$

$\sin x - \frac{\sin^3 x}{3} + C$

or

$\rightarrow \int (\cos x - \sin^2 x \cos x)\,dx$

$\int \cos x\,dx - \int \sin^2 x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\sin x - \int u^2\,du$

$\sin x - \frac{u^3}{3} + C$

$\sin x - \frac{\sin^3 x}{3} + C$

$\sin x - \frac{\sin^3 x}{3} + C$