Name: _____ Date: _____

AP Calc AB: Some Special Antiderivatives Ms. Loughran

Do Now:

why $|x|$?

$[\ln x]' = \frac{1}{x} \cdot 1 = \frac{1}{x}$

$[\ln(-x)]' = \frac{1}{-x} \cdot -1 = \frac{1}{x}$

1. $\int \frac{1}{x} dx =$ $\ln|x| + C$

2. $\int e^x dx =$ $e^x + C$

3. $\int \cos x dx =$ $\sin x + C$

4. $\int \sin x dx =$ $-\cos x + C$

Special Antiderivatives

$\int \frac{1}{x} dx =$ $\ln|x| + C$

$\int e^x dx =$ $e^x + C$

$\int \cos x dx =$ $\sin x + C$

$\int \sin x dx =$ $-\cos x + C$

Homework 01-29

(7) a) $\int x^8\,dx = \frac{1}{9}x^9 + C$

b) $\int x^{5/7}\,dx = 7/12\,x^{12/7} + C$

c) $\int x^3\sqrt{x}\,dx$

$\int x^3 \cdot x^{\frac{1}{2}}\,dx = \int x^{7/2}\,dx$

$2/9\,x^{9/2} + C$

8 a) $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx$

$\frac{3}{5}x^{5/3} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$

b) $\int \frac{1}{x^6}\,dx = \int x^{-6}\,dx$

$-\frac{1}{5}x^{-5} + C$

$\frac{-1}{5x^5} + C$

c) $\int x^{-7/8}\,dx$

$8x^{\frac{1}{8}} + C$

(9) (a) $\int \frac{1}{2x^3}\,dx$

$\frac{1}{2}\int x^{-3}\,dx$

$\frac{1}{2}\,\frac{x^{-2}}{-2}$

$-\frac{1}{4}x^{-2} + C = \frac{-1}{4x^2} + C$

(b) $\int (u^3 - 2u + 7)\,du$

$\int u^3\,du - 2\int u\,du + \int 7\,du$

$\frac{1}{4}u^4 - u^2 + 7u + C$

(10) $\int (x^{2/3} - 4x^{-\frac{1}{5}} + 4)\,dx$

$\frac{3}{5}x^{5/3} - \frac{4.5}{4}x^{\frac{4}{5}} + 4x + C$

$\frac{3}{5}x\sqrt[3]{x^2} - 5\sqrt[5]{x^4} + 4x + C$

(11) $\int (x^{-3} + \sqrt{x} - 3x^{\frac{1}{4}} + x^2)\,dx$

$\int x^{-3}\,dx + \int x^{\frac{1}{2}}\,dx - 3\int x^{\frac{1}{4}}\,dx + \int x^2\,dx$

$-\frac{1}{2x^2} + \frac{2}{3}x^{3/2} - 12/5\,x^{5/4} + \frac{1}{3}x^3 + C$

(12) $\int (7y^{-3/4} - y^{\frac{1}{3}} + 4y^{\frac{1}{2}})\,dy$

$7 \cdot 4\,y^{\frac{1}{4}} - \frac{3}{4}y^{\frac{4}{3}} + 4 \cdot \frac{2}{3}y^{3/2} + C$

$28y^{\frac{1}{4}} - \frac{3}{4}y^{\frac{4}{3}} + \frac{8}{3}y^{3/2} + C$

(13) $\int x(1 + x^3)\,dx = \int (x + x^4)\,dx = \frac{1}{2}x^2 + \frac{1}{5}x^5 + C$

(14) $\int (2+y^2)^2\,dy$

$\int (4 + 4y^2 + y^4)\,dy$

$4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + C$

(15) $\int x^{\frac{1}{3}}(2-x)^2\,dx = \int (4x^{\frac{1}{3}} - 4x^{4/3} + x^{7/3})\,dx = 3x^{4/3} - 12/7\,x^{7/3} + 3/10\,x^{10/3} + C$

$4-4x+x^2$

$\frac{4\cdot 3}{4}x^{\frac{4}{3}} - 4 \cdot \frac{3}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C$

(17) $\int \frac{x^5 + 2x^2 - 1}{x^4}\,dx = \int (x + 2x^{-2} - x^{-4})\,dx = \frac{1}{2}x^2 - \frac{2}{x} + \frac{1}{3x^3} + C$

$\frac{x^{-1}}{-1}$ $\frac{x^{-3}}{-3}$

$\frac{-1}{x}$

(16) $\int (1+x^3)(2-x)\,dx$

$\int (2 + 2x^2 - x - x^3)\,dx = 2x + \frac{2}{3}x^3 - \frac{1}{2}x^2 - \frac{x^4}{4} + C$

Please complete numbers 1-14, 16, 19, 21-32, 41-48, 50, 52, 54, 55, 59

Exercises and Problems

Exercises

In Exercises 1–21, find an antiderivative.

1. $f(x) = 5$
2. $f(x) = 5x$
3. $f(x) = x^2$
4. $g(t) = t^2 + t$
5. $h(t) = \cos t$
6. $g(z) = \sqrt{z}$
7. $h(z) = \frac{1}{z}$
8. $r(t) = \frac{1}{t^2}$
9. $g(z) = \frac{1}{z^3}$
10. $f(z) = e^z$
11. $g(t) = \sin t$
12. $f(t) = 2t^2 + 3t^3 + 4t^4$
13. $p(t) = t^3 - \frac{t^2}{2} - t$
14. $q(y) = y^4 + \frac{1}{y}$
15. $p(t) = \cos t + \frac{1}{\cos^2 t}$
16. $f(t) = \frac{t^2+1}{t}$
17. $p(\theta) = 2\sin(2\theta)$
18. $r(t) = e^t + 5e^{5t}$
19. $q(t) = (t+1)^2$
20. $f(x) = 5^x$
21. $f(x) = 5x - \sqrt{x}$

In Exercises 22–32, find the general antiderivative.

22. $f(t) = 6t$
23. $h(x) = x^3 - x$
24. $f(x) = x^2 - 4x + 7$
25. $r(t) = t^3 + 5t - 1$
26. $f(z) = z + e^z$
27. $g(t) = \sqrt{t}$
28. $g(x) = \sin x + \cos x$
29. $h(x) = 4x^3 - 7$
30. $p(t) = 2 + \sin t$
31. $p(t) = \frac{1}{\sqrt{t}}$
32. $g(x) = \frac{5}{x^3}$

In Exercises 33–40, find an antiderivative $F(x)$ with $F'(x) = f(x)$ and $F(0) = 0$. Is there only one possible solution?

33. $f(x) = 3$
34. $f(x) = 2x$
35. $f(x) = -7x$
36. $f(x) = \frac{1}{4}x$
37. $f(x) = x^2$
38. $f(x) = \sqrt{x}$
39. $f(x) = 2 + 4x + 5x^2$
40. $f(x) = \sin x$

Find the indefinite integrals in Exercises 41–59.

41. $\int 5x\,dx$
42. $\int x^3\,dx$
43. $\int \sin\theta\,d\theta$
44. $\int (x^3 - 2)\,dx$
45. $\int \left(t^2 + \frac{1}{t^2}\right) dt$
46. $\int 4\sqrt{w}\,dw$
47. $\int (x^2 + 5x + 8)\,dx$
48. $\int \frac{4}{t^2}\,dt$
49. $\int (4t + 7)\,dt$
50. $\int \cos\theta\,d\theta$
51. $\int 5e^z\,dz$
52. $\int \left(x + \frac{1}{\sqrt{x}}\right) dx$
53. $\int \sin t\,dt$
54. $\int (\pi + x^{11})\,dx$
55. $\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}}\right) dt$
56. $\int \cos(x+1)\,dx$
57. $\int e^{2r}\,dr$
58. $\int \frac{1}{e^z}\,dz$
59. $\int \left(\frac{y^2-1}{y}\right)^2 dy$

In Exercises 60–71, evaluate the definite integrals exactly [as in $\ln(3\pi)$], using the Fundamental Theorem, and numerically [$\ln(3\pi) \approx 2.243$]:

60. $\int_0^3 (x^2 + 4x + 3)\,dx$
61. $\int_1^3 \frac{1}{t}\,dt$
62. $\int_0^{\pi/4} \sin x\,dx$
63. $\int_0^2 3e^x\,dx$
64. $\int_2^5 (x^3 - \pi x^2)\,dx$
65. $\int_0^1 \sin\theta\,d\theta$
66. $\int_1^2 \frac{1+y^2}{y}\,dy$
67. $\int_0^2 \left(\frac{x^3}{3} + 2x\right) dx$
68. $\int_0^{\pi/4} (\sin t + \cos t)\,dt$
69. $\int_0^1 2e^x\,dx$
70. $\int_{-3}^{-1} \frac{2}{r^3}\,dr$
71. $\int_0^{\pi/4} \frac{1}{\cos^2 x}\,dx$