Name: ______________________________________ Date: _____________

AP Calc AB:More Special Antiderivatives Ms. Loughran

Do Now:

1. $\int \sec^2 x\, dx =$ tan x + c

2. $\int \cot x \csc x\, dx =$ − csc x + c

3. $\int \tan x \sec x\, dx =$ sec x + c

4. $\int \csc^2 x\, dx =$ − cot x + c

More Special Antiderivatives

$\int \sec^2 x\, dx =$ tan x + c

$\int \cot x \csc x\, dx =$ − csc x + c

$\int \tan x \sec x\, dx =$ sec x + c

$\int \csc^2 x\, dx =$ − cot x + c

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REWRITING THE INTEGRAND

We learned that, by rewriting, we can sometimes avoid use of the Quotient Rule for differentiation. Similarly, you should learn that one of the most important steps in integration is REWRITING THE INTEGRAND in a form that fits the basic integration rules. Doing this can help you do many problems more easily.

DIRECTIONS: Several examples of rewriting the integrand are shown below. Read these and then finish the problems that are not completed.

FUNCTION	REWRITE	INTEGRATE	SIMPLIFY
1. $\int \frac{2}{\sqrt{x}}\,dx$	$2\int x^{-\frac{1}{2}}\,dx$	$2\left(\frac{x^{\frac{1}{2}}}{1/2}\right) + C$	$4x^{\frac{1}{2}} + C$ or $4\sqrt{x} + C$
2. $\int (y^2 - 1)^2\,dy$	$\int(y^4-2y^2+1)dy$	$\frac{y^5}{5} - 2\left(\frac{y^3}{3}\right) + y + C$	$\frac{1}{5}y^5 - \frac{2}{3}y^3 + y + C$
3. $\int \frac{x^3+3}{x^2}\,dx$ $\int\left(\frac{x^3}{x^2} + \frac{3}{x^2}\right)dx$	$\int(x + 3x^{-2})dx$	$\frac{x^2}{2} + 3\left(\frac{x^{-1}}{-1}\right) + C$	$\frac{1}{2}x^2 - \frac{3}{x} + C$
4. $\int \sqrt[3]{x}(x-4)\,dx$ $\int\left(x^{\frac{1}{3}}(x-4)\right)dx$	$\int(x^{4/3} - 4x^{1/3})dx$	$\frac{x^{7/3}}{7/3} - 4\left(\frac{x^{4/3}}{4/3}\right) + C$	$\frac{3}{7}x^{4/3}(x-7) + C$
5. $\int 2\sqrt[3]{x^2}\,dx$	$= \int 2x^{2/3}dx$	$2 \cdot \frac{3}{5}x^{5/3} + c$	$\frac{6}{5}\sqrt[3]{x^5} + c$ $\frac{6}{5}x\sqrt[3]{x^2} + c$
6. $\int \frac{2}{x^2}\,dx$	$= \int 2x^{-2}dx$	$2 \cdot \frac{x^{-1}}{-1} + c$	$-\frac{2}{x} + c$
7. $\int \frac{1}{x\sqrt{x}}\,dx$	$= \int \frac{1}{x^{3/2}}dx$	$\int x^{-3/2}dx$ $-2x^{-\frac{1}{2}} + c$	$\frac{-2}{\sqrt{x}} + c$
8. $\int x(x^3 - 4)\,dx$	$\int(x^4 - 4x)\,dx$	$\frac{x^5}{5} - 4\frac{x^2}{2} + c$	$\frac{1}{5}x^5 - 2x^2 + c$
9. $\int \frac{1}{2x^3}\,dx$	$\frac{1}{2}\int x^{-3}dx$	$\frac{1}{2} \cdot \frac{x^{-2}}{-2} + c$	$-\frac{1}{4x^2} + c$
10. $\int \frac{1}{(2x)^3}\,dx$	$= \int \frac{1}{8x^3}dx = \frac{1}{8}\int x^{-3}dx$	$\frac{1}{8} \cdot \frac{x^{-2}}{-2} + c$	$\frac{-1}{16x^2} + c$
11. $\int (2 - y^2)^2\,dy$	$= \int(4 - 4y^2 + y^4)\,dy$	$4y - \frac{4y^3}{3} + \frac{y^5}{5} + c$	$4y - \frac{4}{3}y^3 + \frac{1}{5}y^5 + c$
12. $\int \frac{x^2+1}{x^2}\,dx$	$= \int(1 + x^{-2})dx$	$x + \frac{x^{-1}}{-1} + c$	$x - \frac{1}{x} + c$
13. $\int \frac{dy}{\csc y}$	$= \int \sin y\,dy$	$-\cos y + c$	
14. $\int \frac{\sin 2x}{\cos x}\,dx$	$= \int \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}}dx$	$\int 2\sin x\,dx$	$-2\cos x + c$
15. $\int (4\sec^2 x + \csc x \cot x)\,dx$ $4\int \sec^2 x\,dx + \int \csc x \cot x\,dx$		$4\tan x - \csc x + c$	

Classwork 01-30

Integration

Exercises + Problems Key

1. $F(x) = 5x$
2. $F(x) = \frac{5}{2}x^2$
3. $F(x) = \frac{x^3}{3}$
4. $G(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2$
5. $H(t) = \sin t$
6. $G(z) = \frac{2}{3}\sqrt{z^3}$
7. $H(z) = \ln|z|$
8. $R(t) = -\frac{1}{t}$
9. $G(x) = -\frac{1}{2z^2}$
10. $F(z) = e^z$
11. $G(t) = -\cos t$
12. $F(t) = \frac{2}{3}t^3 + \frac{3}{4}t^4 + \frac{4}{5}t^5$
13. $P(t) = \frac{1}{4}t^4 - \frac{1}{6}t^3 - \frac{1}{2}t^2$
14. $q(y) = \frac{1}{5}y^5 + \ln|y|$

16. $F(t) = \frac{1}{2}t^2 + \ln|t|$

19. $Q(t) = \frac{1}{3}t^3 + t^2 + t$

21. $F(x) = \frac{5}{2}x^2 - \frac{2}{3}x^{3/2}$
22. $3t^2 + C$
23. $\frac{1}{4}x^4 - \frac{1}{2}x^2 + C$
24. $\frac{1}{3}x^3 - 2x^2 + 7x + C$
25. $\frac{1}{4}t^4 + \frac{5}{2}t^2 - t + C$
26. $\frac{1}{2}z^2 + e^z + C$
27. $\frac{2}{3}t^{3/2} + C$
28. $-\cos x + \sin x + C$
29. $x^4 - 7x + C$
30. $2t - \cos t + C$
31. $2\sqrt{t} + C$
32. $\frac{-5}{2x^2} + C$

41. $\frac{5x^2}{2} + C$
42. $\frac{1}{4}x^4 + C$
43. $-\cos\theta + C$
44. $\frac{1}{4}x^4 - 2x + C$
45. $\frac{1}{3}t^3 - \frac{1}{t} + C$
46. $\frac{8}{3}w^{3/2} + C$
47. $\frac{1}{3}x^3 + \frac{5}{2}x^2 + 8x + C$
48. $\frac{-4}{t} + C$

50. $\sin\theta + C$

52. $\frac{1}{2}x^2 + 2\sqrt{x} + C$

54. $\pi x + \frac{1}{12}x^{12} + C$
55. $\frac{2}{5}t^{5/2} - 2t^{-\frac{1}{2}} + C$

59. $\frac{1}{3}y^3 - 2y - \frac{1}{y} + C$

Homework 01-30

① $3x^2 + x\sec^2 y \, dy/dx + \tan y = 0$

$dy/dx \, x\sec^2 y = -3x^2 - \tan y$

$\frac{dy}{dx} = \frac{-3x^2 - \tan y}{x\sec^2 y}$

$\left.\frac{dy}{dx}\right|_{(3,0)} = \frac{-3(3)^2 - \tan 0}{3\sec^2(0)} = \frac{-27}{3} = -9$

$y = -9(x-3)$ $\quad y = -9(3.\overset{.1}{1}-3) = -.9$ (B).

② $(2,5)$

$G'(2) = \frac{10(2)}{9-2^2} = \frac{20}{5} = 4$

$y - 5 = 4(x-2)$

$y = 4(x-2) + 5$

$y = 4(2.2-2) + 5 = 4(.2) + 5 = 5.8$ (C).

③ $f(x) = x^2 \sin(\pi x) - 3x$ $\quad f(1) = 1\sin(\overset{0}{\pi}) - 3 = -3$

$f'(x) = 2x\sin(\pi x) + x^2\cos(\pi x)\cdot\pi - 3$

$f'(1) = 2(1)\sin(\pi) + 1^2\cos(\pi)\cdot\pi - 3$

$2\overset{0}{\sin}\pi + \pi\overset{-1}{\cos}(\pi) - 3 = -\pi - 3$

$y + \underset{-3}{3} = (-\pi-3)(x-1) = -\pi x - 3x + \pi + \underset{-3}{\cancel{3}}$

$y = -\pi x - 3x + \pi = (-\pi-3)x + \pi$ (D)

$\sqrt{3}\int x\sqrt{x}\,dx$

(4) $\int x\sqrt{3x}\,dx = \int x\cdot 3^{1/2}\cdot x^{1/2}\,dx = 3^{1/2}\int x^{3/2}\,dx$

$\sqrt{3}\cdot\frac{2}{5}x^{5/2}+C$

$\frac{2\sqrt{3}}{5}x^{5/2}+C$ (A)

(5) $\int e^{4-\ln x}\,dx = \int e^{4}\div e^{\ln x}\,dx = e^{4}\int \frac{1}{x}\,dx = e^{4}\ln|x|+C$ (E)

$\int \frac{e^{4}}{x}\,dx$

$\int (e^{4}\cdot e^{-\ln x})\,dx = \int (e^{4}\cdot e^{\ln x^{-1}})\,dx$

$\int e^{4}\cdot x^{-1}\,dx$

$e^{4}\int x^{-1}\,dx$