

Name: _____
AP Calculus AB: Integrating Using U-Substitution

Date: _____
Ms. Loughran

Do Now:

$$1. \int \frac{dt}{t\sqrt{2t}} = \frac{1}{\sqrt{2}} \int \frac{dt}{t\sqrt{t}} = \frac{1}{\sqrt{2}} \int t^{-3/2} dt = \frac{1}{\sqrt{2}} \cdot -2t^{-1/2} + C$$
$$\frac{-2}{\sqrt{2t}} + C$$

Classwork:

1. $\int 2x(x^2+1)^{23} dx$

$$u = x^2 + 1$$
$$du = 2x dx$$

$$\int u^{23} du$$

$$\frac{1}{24} u^{24} + C$$

$$\frac{1}{24} (x^2+1)^{24} + C$$

2. $\int \frac{3x dx}{\sqrt{4x^2+5}}$

$$u = 4x^2 + 5$$

$$du = 8x dx$$

$$\frac{du}{8} = x dx$$

$$\int 3 \cdot \frac{du}{8} \cdot u^{-1/2}$$

$$\frac{3}{8} \int u^{-1/2} du$$

$$\frac{3}{4 \cdot 8} \cdot 2u^{1/2} + C$$

$$\frac{3}{4} (4x^2+5)^{1/2} + C$$

$$\frac{3}{4} \sqrt{4x^2+5} + C$$

Steps:

- find u:
1. look for a piece of the original integrand whose derivative is also in the integrand (try denominator, anything raised to a power, anything under a radical)

2. set $u =$ that piece, take the derivative of u with respect to the variable in the expression

3. use u and du to replace the original integral and your new integral should be easier to work with

4. integrate

5. replace u (don't forget $+C$ if it's an indefinite integral)

$$3. \int \frac{x^2}{x^3-4} dx$$

$$u = x^3 - 4$$

$$du = 3x^2 dx$$

$$\frac{du}{3} = x^2 dx$$

$$\frac{1}{3} \int \frac{1}{u} du$$

$$\frac{1}{3} \ln |u| + C$$

$$\frac{1}{3} \ln |x^3 - 4| + C$$

$$4. \int x(2-x^2)^3 dx$$

$$u = 2 - x^2$$

$$du = -2x dx$$

$$\frac{du}{-2} = x dx$$

$$-\frac{1}{2} \int u^3 du$$

$$-\frac{1}{2} \cdot \frac{u^4}{4} + C$$

$$-\frac{1}{8} (2-x^2)^4 + C$$

$$5. \int \sin x e^{\cos x} dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$-du = \sin x dx$$

$$-\int e^u du = -e^u + C$$

$$-e^{\cos x} + C$$

Last night's hw was a DeltaMath assignment.