Name: _____ Date: _____

AP Calculus AB: More Integration with Trig Functions Ms. Loughran

Do Now:

1. $\int \frac{t+1}{t}\,dt =$ $\int\left(1+\frac{1}{t}\right)dt = t + \ln|t| + C$

2. $\int \cos(3x)\,dx =$ $\frac{1}{3}\sin(3x) + C$

3. $\int \frac{\sin x}{\cos x}\,dx =$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$-\int \frac{1}{u}\,du = -\ln|u| + C$

$-\ln|\cos x| + C$

or $\int \tan x\,dx = -\ln|\cos x| + C$

Classwork

4. $\int \frac{\cos x}{\sin x}\,dx =$

$u = \sin x$

$du = \cos x\,dx$

$\int \frac{1}{u}\,du = \ln|u| + C$

$= \ln|\sin x| + C$

or $\int \cot x\,dx = \ln|\sin x| + C$

$\int \tan x\,dx = -\ln|\cos x| + C$ $\int \cot x\,dx = \ln|\sin x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$ $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

5. $\int \frac{\cos x}{\sin^2 x}\,dx =$ * $\int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\,dx$

* $\int \cot x \csc x\,dx = -\csc x + C$

or

$u = \sin x$

$du = \cos x\,dx$

$\int u^{-2}\,du = -u^{-1} + C$

$-\frac{1}{\sin x} + C$

6. $\int \sec^3(2x)\tan(2x)dx =$

$\int \sec^2(2x)\sec(2x)\tan(2x)\,dx$

$u = \sec(2x)$

$du = 2\tan(2x)\sec(2x)\,dx$

$\frac{du}{2} = \tan(2x)\sec(2x)\,dx$

$\frac{1}{2}\int u^2\,du = \frac{1}{2}\cdot\frac{1}{3}u^3 + C$

$\frac{1}{6}(\sec(2x))^3 + C$

$\frac{1}{6}\sec^3(2x) + C$

7. $\int \cot(7x)dx =$

$\frac{1}{7}\ln|\sin(7x)| + C$ or $\int \frac{\cos(7x)}{\sin(7x)}\,dx$

$u = \sin(7x)$

$du = 7\cos(7x)\,dx$

$\frac{du}{7} = \cos(7x)\,dx$

$\frac{1}{7}\int \frac{1}{u}\,du = \frac{1}{7}\ln|u| + C$

$\frac{1}{7}\ln|\sin(7x)| + C$

8. $\int \frac{dx}{\cos^2(2x)} =$ $\int \sec^2(2x)\,dx = \frac{1}{2}\tan(2x) + C$

9. $\int \cot^2(3x)dx =$

$\int (\csc^2(3x) - 1)\,dx$

$-\frac{1}{3}\cot(3x) - x + C$

Pythagorean Id:

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

10. $\int \cos^3 x dx =$

$\int \cos^2 x \cdot \cos x\,dx$

$\int (1 - \sin^2 x)\cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int (1 - u^2)\,du = u - \frac{u^3}{3} + C$

$\sin x - \frac{\sin^3 x}{3} + C$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

More Practice

1. $\int \cos(8x)dx =$

2. $\int \frac{5}{1+x^2}dx =$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$5\int \frac{1}{1+x^2}\,dx$

$5\arctan x + C$

or $5\tan^{-1}x + C$

3. $\int \frac{1}{\sqrt{1-x^2}}dx =$

4. $\int \sqrt{x-1}\sqrt{x+1}\,x\,dx =$

5. $\int \sqrt{3+x^2}\,x^3 dx =$

6. $\int (e^{2x}+1)e^{-x}dx =$

7. $\int x^2 \sec^2(x^3)dx =$

8. $\int \cos(4\theta)\sqrt{2-\sin(4\theta)}d\theta =$

Homework 02-02

Practice With Antiderivatives Answer Key

① $\int (3+x)^2\,dx$

$u = 3+x$

$du = dx$

$\int u^2\,du$

$= u^3/3 + C = \frac{1}{3}(3+x)^3 + C$

$\int (9+6x+x^2)\,dx$ or

$9x + 6\cdot\frac{x^2}{2} + \frac{x^3}{3} + C$

$9x + 3x^2 + \frac{x^3}{3} + C$

$\frac{1}{3}x^3 + 3x^2 + 9x + C$

② $\int \frac{x+3x^2}{\sqrt{x}}\,dx$

$x^{\frac{1}{2}}$

$\int (x^{\frac{1}{2}} + 3x^{3/2})\,dx$

$\frac{2}{3}x^{3/2} + 3\left(\frac{2}{5}x^{5/2}\right) + C$

$2/3\,x^{3/2} + 6/5\,x^{5/2} + C$

③ $\int \frac{x\,dx}{(x^2+1)^3}$

$u = x^2+1$

$du = 2x\,dx$

$du/2 = x\,dx$

$\frac{1}{2}\int u^{-3}\,du = \frac{1}{2}\,\frac{u^{-2}}{-2} + C$

$= -\frac{1}{4(x^2+1)^2} + C$

④ $\int \sec^2 x\,dx = \tan x + C$

⑤ $\int \frac{dx}{\cos^2(5x)} = \int \sec^2(5x)\,dx$

let $u = 5x$

$du = 5\,dx$

$du/5 = dx$

$\frac{1}{5}\int \sec^2 u\,du$

$\frac{1}{5}\tan u + C$

$\frac{1}{5}\tan(5x) + C$

⑥ $\int \tan x \sec^2 x\,dx$

$\int u\,du$

$\frac{u^2}{2} + C = \frac{\tan^2 x}{2} + C$

or $u = \sec x$

$du = \tan x \sec x\,dx$

$\frac{\sec^2 x}{2} + C$

$u = \tan x$

$du = \sec^2 x\,dx$

$\frac{\sec^2 x}{2} + C$

$\frac{1}{2\cos^2 x} + C$

$\frac{1}{2u^2} + C$

or $\int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos^2 x}\,dx$

$u = \cos x$

$du = -\sin x$

$-\int u^{-3}\,du = \frac{-u^{-2}}{-2}$

$\int \frac{\sin x}{\cos^3 x}\,dx$

OR $\int \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}\, d\theta = \int \tan\theta \sec\theta\, d\theta = \sec\theta + C$

⑦ $\int \frac{\sin\theta\, d\theta}{\cos^2\theta}$

let $u = \cos\theta$

$du = -\sin\theta\, d\theta$

$-du = \sin\theta\, d\theta$

$-\int u^{-2}\, du = -\frac{u^{-1}}{-1} + C$

$= \frac{1}{\cos\theta} + C = \sec\theta + C$

⑧ $\int \sin 3\theta\, d\theta$

$u = 3\theta$

$du = 3\, d\theta$

$du/3 = d\theta$

$\frac{1}{3}\int \sin u\, du$

$-\frac{1}{3}\cos u + C$

$-\frac{1}{3}\cos 3\theta + C$

⑨ $\int \frac{2^{\sqrt{t}}}{\sqrt{t}}\, dt$

$u = \sqrt{t} = t^{1/2}$

(2) $du = \frac{1}{2} \cdot t^{-1/2} =$ (2) $\frac{1}{2\sqrt{t}}$

$2du = \frac{1}{\sqrt{t}}\, dt$

$2\int e^u\, du = 2e^u + C$

$= 2e^{\sqrt{t}} + C$

⑩ $\int \sin x \cos x\, dx$

$u = \sin x$

$du = \cos x\, dx$

$\int u\, du$

$\frac{u^2}{2} + C$

OR

$u = \cos x$

$du = -\sin x\, dx$

$-\int u\, du$

$\frac{\sin^2 x}{2} + C$

$-\frac{u^2}{2} + C = -\frac{\cos^2 x}{2} + C$

Trig window
Ymin -2
Ymax 2

same graph differs by a constant