Name: ______________________________ Date: __________

AP Calc AB: Optimization Ms. Loughran

Do Now

1. A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$6000?

500 ft by $\frac{6000 - 6(500)}{4}$ ft

$A = lw$

$\boxed{P = 2l + 2w}$

Rest: $l > 0$

$1500 - \frac{3}{2}l > 0$

$\frac{2}{3} \cdot 1500 > \frac{3}{2}l \cdot \frac{2}{3}$

$1000 > l$

$0 < l < 1000$

$A(l) = l \cdot \left(1500 - \frac{3}{2}l\right)$

$A(l) = 1500l - \frac{3}{2}l^2$

$A'(l) = 1500 - 3l$

$0 = 1500 - 3l$

$3l = 1500$

$l = 500$

$A''(l) = -3$

$\text{Cost} = 3 \cdot 2l + 2 \cdot 2w$

$\text{Cost} = 6l + 4w$

$6000 = 6l + 4w$

$\frac{6000 - 6l}{4} = w$

$w = 1500 - \frac{3}{2}l$

2. A cylindrical can is to have a volume of 400 cubic centimeters. Find the dimensions of the can so as to minimize its total surface area.

$\sqrt[3]{\frac{200}{\pi}}$ cm by $\frac{400}{\pi\left(\sqrt[3]{\frac{200}{\pi}}\right)^2}$ cm

$V = 400\text{cm}^3$

$\pi r^2 h = 400$

$h = \frac{400}{\pi r^2}$

$r > 0$

$SA = 2\pi r^2 + 2\pi r h$

$SA(r) = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\frac{400}{\cancel{\pi} r^{\cancel{2}}}\right)$

$SA(r) = 2\pi r^2 + 800r^{-1}$

$SA'(r) = 4\pi r - 800r^{-2}$

$0 = 4\pi r - \frac{800}{r^2}$

$\frac{800}{r^2} = 4\pi r$

$4\pi r^3 = 800$

$r^3 = \frac{200}{\pi}$

$r = \sqrt[3]{\frac{200}{\pi}}$ cm

$SA''(r) = 4\pi + 1600r^{-3} > 0$

Homework 02-05

More Practice

1. $\int \cos(8x)dx =$

$u = 8x$

$du = 8dx$

$\frac{du}{8} = dx$

$\frac{1}{8}\int \cos u \, du = \frac{1}{8}\sin u + C$

$\frac{1}{8}\sin(8x) + C$

2. $\int \frac{5}{1+x^2}dx =$

$5\int \frac{1}{1+x^2}dx$

$5\tan^{-1}x + C$

3. $\int \frac{1}{\sqrt{1-x^2}}dx =$

$\sin^{-1}(x) + C$

$\arcsin(x) + C$

4. $\int \sqrt{x-1}\sqrt{x+1}\, x\,dx$

$u = x^2 - 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \sqrt{x^2-1}\, x\,dx$

$\frac{1}{2}\int u^{\frac{1}{2}} du$

$\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$\frac{1}{3}(x^2-1)^{3/2} + C$

5. $\int \sqrt{3+x^2}\,x^3 dx =$

$x^2 x\,dx$

$u = 3 + x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$u = 3 + x^2$

$u - 3 = x^2$

$\frac{1}{2}\int u^{\frac{1}{2}}\cdot(u-3)\,du$

$\frac{1}{2}\int (u^{3/2} - 3u^{\frac{1}{2}})\,du$

$\frac{1}{2}\left(\frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C\right)$

$\frac{1}{5}u^{5/2} - u^{3/2} + C = \frac{1}{5}(3+x^2)^{5/2} - (3+x^2)^{3/2} + C$

6. $\int (e^{2x}+1)e^{-x}dx =$

$\int (e^x + e^{-x})\,dx$

$\int e^x dx + \int e^{-x}dx$

$e^x - e^{-x} + C$

$\int e^u du$

$-e^u + C$

$u = -x$

$du = -dx$

$-du = dx$

7. $\int x^2 \sec^2(x^3)dx =$

$u = x^3$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$\frac{1}{3}\int \sec^2 u \, du$

$\frac{1}{3}\tan u + C = \frac{1}{3}\tan(x^3) + C$

8. $\int \cos(4\theta)\sqrt{2-\sin(4\theta)}d\theta =$

$u = 2 - \sin(4\theta)$

$du = -4\cos(4\theta)\,d\theta$

$-\frac{du}{4} = \cos(4\theta)\,d\theta$

$-\frac{1}{4}\int u^{\frac{1}{2}} du$

$-\frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{6}(2-\sin 4\theta)^{3/2} + C$

More Practice

1. $\int \cos(8x)dx =$

2. $\int \frac{5}{1+x^2}dx =$

3. $\int \frac{1}{\sqrt{1-x^2}}dx =$

4. $\int \sqrt{x-1}\sqrt{x+1}\; x\,dx =$

5. $\int \sqrt{3+x^2}\, x^3 dx =$

$x^2\, x\,dx$

need in terms of u

$u = 3 + x^2$

$u - 3 = x^2$

$u = 3 + x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int u^{\frac{1}{2}}(u-3)\,du$

$\frac{1}{2}\int \left(u^{3/2} - 3u^{\frac{1}{2}}\right)du = \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \cancel{3}\cdot\frac{2}{\cancel{3}}u^{3/2}\right] + C$

$\frac{1}{5}(3+x)^{5/2} - (3+x)^{3/2} + C$

6. $\int (e^{2x}+1)e^{-x}dx =$

7. $\int x^2 \sec^2(x^3)dx =$

8. $\int \cos(4\theta)\sqrt{2-\sin(4\theta)}\,d\theta =$