Do Now: Pick a partner and work on this page- 15 min

Name: ______________________________ Date: _____
AP Calculus Properties of Definite Integrals Intro Ms. Loughran

Find each of the following integrals.

1. $\int_{2}^{2}(x^2+1)dx =$ 0
   $\frac{x^3}{3}+x \Big|_{2}^{2}$

2. $\int_{-1}^{-1}(x-2)dx =$ 0
   $\frac{x^2}{2}-2x \Big|_{-1}^{1}$

3. $\int_{-1}^{1}(x^2+1)dx =$ $\frac{8}{3}$

4. $\int_{1}^{2}(x^2+1)dx =$ $\frac{10}{3}$

5. $\int_{-1}^{2}(x^2+1)dx =$ $\frac{18}{3} = 6$

6. $\int_{2}^{-1}(x^2+1)dx =$ $\frac{-18}{3} = -6$

7. $\int_{0}^{2}x^2dx =$ $\frac{8}{3}$
   even
   $\frac{x^3}{3}\Big|_{0}^{2}$

8. $\int_{-2}^{0}x^2dx =$ $\frac{8}{3}$

9. $\int_{-2}^{2}x^2dx =$ $\frac{16}{3}$

10. $\int_{0}^{2}x^3dx =$ 4
    odd
    $\frac{x^4}{4}\Big|_{0}^{2}$

11. $\int_{-2}^{0}x^3dx =$ -4

12. $\int_{-2}^{2}x^3dx =$ 0

Now see if you can answer the following questions.

1. $\int_{a}^{a}f(x)dx =$ 0

2. Rewrite $\int_{a}^{b}f(x)dx+\int_{b}^{c}f(x)dx$ using a single integral. $\int_{a}^{c}f(x)dx$

3. What is the relationship between $\int_{a}^{b}f(x)dx$ and $\int_{b}^{a}f(x)dx$? $\int_{a}^{b}f(x)dx = -\int_{b}^{a}f(x)dx$

4. If $f(x)$ is an even function, fill in the following blank: $\int_{-a}^{a}f(x)dx =$ <u>2</u> $\int_{0}^{a}f(x)dx$ or $2\int_{-a}^{0}f(x)dx$

5. If $f(x)$ is an odd function, fill in the following blank: $\int_{-a}^{a}f(x)dx =$ <u>0</u>

even: $f(-x) = f(x)$

odd: $f(-x) = -f(x)$

remember

$\int_{0}^{a}$ odd function is the opposite of $\int_{-a}^{0}$ of that same odd function

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USING INTEGRATION THEOREMS

1. Use the fact that

even

$$\int_0^2 x^2\,dx = \frac{8}{3}$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^2\,dx$ $\frac{8}{3}$

(b) $\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2 dx = 2\left(\frac{8}{3}\right) = \frac{16}{3}$

(c) $\int_0^2 -x^2\,dx = -\int_0^2 x^2 dx = \frac{-8}{3}$

(d) $\int_0^2 (x^2+1)\,dx$

$\int_0^2 x^2 dx + \int_0^2 1\,dx$

$\frac{8}{3} + x\Big|_0^2$

$\frac{8}{3} + (2-0) = \frac{8}{3} + 2 = \frac{14}{3}$

(e) $\int_{-2}^{0} 3x^2\,dx$

$3\int_{-2}^{0} x^2 dx = 3\left(\frac{8}{3}\right) = 8$

2. Use the fact that

odd

$$\int_0^2 x^3\,dx = 4$$

to evaluate the following definite integrals without using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^{0} x^3\,dx = -4$

(b) $\int_{-2}^{2} x^3\,dx = 0$

(c) $\int_0^2 -x^3\,dx = -4$

(d) $\int_0^2 (x^3+1)\,dx = \int_0^2 x^3 dx + \int_0^2 1\,dx$

$4 + x\Big|_0^2$

$4 + (2-0) = 6$

(e) $\int_{-2}^{0} 3x^3\,dx = 3(-4) = -12$

ADDITIVITY

1. If $\int_0^5 f(x)\,dx = 10$ and $\int_5^7 f(x)\,dx = 3$, find

(a) $\int_0^7 f(x)\,dx = 10+3$ $\quad \int_0^5 + \int_5^7 = \int_0^7$

(b) $\int_5^0 f(x)\,dx = -10$

(c) $\int_5^5 f(x)\,dx = 0$

(d) $\int_0^5 3f(x)\,dx = 3\int_0^5 f(x)\,dx = 3(10) = 30$

2. If $\int_0^3 f(x)\,dx = 4$ and $\int_3^6 f(x)\,dx = -1$, find

(a) $\int_0^6 f(x)\,dx = 4+(-1) = 3$

(b) $\int_6^3 f(x)\,dx = -(-1) = 1$

(c) $\int_4^4 f(x)\,dx = 0$

(d) $\int_3^6 -5f(x)\,dx = -5\int_3^6 f(x)\,dx$ $\quad -5(-1) = 5$

3. If $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$, find

(a) $\int_2^6 [f(x) + g(x)]\,dx = \int_2^6 f(x)\,dx + \int_2^6 g(x)\,dx = 10+(-2) = 8$

(b) $\int_2^6 [g(x) - f(x)]\,dx = -2-10 = -12$

(c) $\int_2^6 [2f(x) - 3g(x)]\,dx = 2(10) - 3(-2) = 26$

(d) $\int_2^6 3f(x)\,dx = 3\int_2^6 f(x)\,dx = 3(10) = 30$

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ADDITIVITY OF THE INTEGRAL

(Abbreviated) Theorems: 1) $\int_a^b = \int_a^c + \int_c^b$; and 2) $\int_a^a = 0$; and 3) $\int_a^b = -\int_b^a$.

I: Assume that f has an integral on [1,7] some of whose values are given by: $\int_1^5 f(x)\,dx = 3$, $\int_2^3 f(x)\,dx = 1$, $\int_3^5 f(x)\,dx = 1$ and $\int_3^7 f(x)\,dx = 6$. Evaluate

ANSWERS

1) $\int_1^3 f(x)\,dx$ $\Rightarrow \int_1^3 + \int_3^5 = \int_1^5$ $\quad \int_1^3 + 1 = 3$ $\quad$ 2

2) $\int_2^5 f(x)\,dx$ $\quad \int_2^3 + \int_3^5 = \int_2^5$ $\quad 1+1 = 2$ $\quad$ 2

3) $\int_2^7 f(x)\,dx$

4) $\int_5^7 f(x)\,dx$

5) $\int_1^2 f(x)\,dx$

6) $\int_1^7 f(x)\,dx$

II: Assume that g has an integral on [1,9] some of whose values are given by: $\int_1^4 g(x)\,dx = 1$, $\int_2^4 g(r)\,dr = 2$, $\int_2^6 g(s)\,ds = 0$ and $\int_6^9 g(t)\,dt = 1$. Evaluate

ANSWERS

7) $\int_1^2 g(u)\,du$

8) $\int_2^9 g(v)\,dv$

9) $\int_4^6 g(w)\,dw$

10) $\int_1^9 g(x)\,dx$

11) $\int_4^9 g(y)\,dy$

12) $\int_1^6 g(z)\,dz$

13) $\int_6^4 g(t)\,dt$

Classwork 02-09

⑦ $\int_0^1 (2t-1)^3 dt$

$u = 2t-1$
$du = 2dt$
$\frac{du}{2} = dt$

$\frac{1}{2}\int_{-1}^{1} u^3 du = \frac{1}{2}\cdot\frac{u^4}{4}\Big|_{-1}^{1} = \frac{1}{8}u^4\Big|_{-1}^{1} = \frac{1}{8}\left(1^4 - (-1)^4\right)$

$\frac{1}{8}(1-1) = 0$ (D)

⑧ $\int_4^9 \frac{2+x}{2\sqrt{x}}\,dx = \frac{1}{2}\int_4^9 \left(2x^{-\frac{1}{2}} + x^{\frac{1}{2}}\right)dx$

$\frac{1}{2}\left(2\cdot 2x^{\frac{1}{2}} + \frac{2}{3}x^{3/2}\right)\Big|_4^9$

$\frac{1}{2}\left(4x^{\frac{1}{2}} + \frac{2}{3}x^{3/2}\right)\Big|_4^9$

$2x^{\frac{1}{2}} + \frac{1}{3}x^{3/2}\Big|_4^9$

$2(9)^{\frac{1}{2}} + \frac{1}{3}(9)^{3/2} - \left(2(4)^{\frac{1}{2}} + \frac{1}{3}(4)^{3/2}\right)$

$6 + 9 - \left(4 + \frac{8}{3}\right)$

$6+9-4-\frac{8}{3} = 11-\frac{8}{3} = \frac{33}{3} - \frac{8}{3} = \frac{25}{3}$

(9) $\int_0^1 e^{-x}\,dx$

$u=-x$ $\quad$ UB : -1

$du=-dx$ $\quad$ LB $\quad 0$

$-du=dx$

$$-\int_0^{-1} e^u\,du = -e^u\Big|_0^{-1} = -e^{-1}-(-e^0)$$

$$-\frac{1}{e}+1$$

(10) $\int_0^1 xe^{x^2}\,dx$

$u=x^2$ $\quad$ UB: 1

$du=2x\,dx$ $\quad$ LB: 0

$du/2=x\,dx$

$$\frac{1}{2}\int_0^1 e^u\,du \quad \frac{1}{2}e^u\Big|_0^1 \quad \frac{1}{2}(e^1-e^0)=\frac{1}{2}(e-1)$$

(11) $\int_0^{\pi/4} \sin 2\theta\,d\theta$

$u=2\theta$ $\quad$ UB: $2(\pi/4)=\pi/2$

$du=2d\theta$ $\quad$ LB $\quad 2(0)=0$

$\frac{du}{2}=d\theta$

$$\frac{1}{2}\int_0^{\pi/2} \sin u \quad -\frac{1}{2}\cos u\Big|_0^{\pi/2} \quad -\frac{1}{2}(\cos \pi/2-\cos 0)$$

$$-\frac{1}{2}(0-1)$$

$$-\frac{1}{2}(-1)=\frac{1}{2}$$

(12) $$\int_1^2 \frac{dz}{3-z}$$

$u = 3-z$

$du = -dz$

$-du = dz$

$$-\int_2^1 u^{-1}\,du \quad = -\ln|u| \Big|_2^1$$

$$-\ln 1 - (-\ln 2)$$

$$0 + \ln 2 = \ln 2 \quad (E)$$