Name: ____ Date: ___________

AP Calculus AB: Evaluating Definite Integrals of Piecewise Functions Ms. Loughran

1. Evaluate $\int_0^8 h(x)dx$ if $h(x) = \begin{cases} 4 & , x < 4 \\ x & , x \geq 4 \end{cases}$

$$\int_0^4 4\,dx + \int_4^8 x\,dx$$

$$4x\Big|_0^4 + \frac{x^2}{2}\Big|_4^8$$

$$4(4) - 4(0) + \left(\frac{8^2}{2} - \frac{4^2}{2}\right)$$

$$16 + (32 - 8) = 16 + 24 = 40$$

3. Given $f(x) = \begin{cases} -x^3 & , x < -2 \\ -3x+2 & , -2 \le x < \pi \\ \sin x & , x \ge \pi \end{cases}$

Find : (a) $\int_{-3}^{0} f(x)\,dx$

$$\int_{-3}^{-2} -x^3\,dx + \int_{-2}^{0} (-3x+2)\,dx$$

$$-\int_{-3}^{-2} x^3\,dx + \int_{-2}^{0} (-3x+2)\,dx$$

$$\int_{-2}^{-3} x^3\,dx + \int_{-2}^{0} (-3x+2)\,dx$$

$$\frac{x^4}{4}\Big|_{-2}^{-3} + \frac{-3x^2}{2} + 2x\Big|_{-2}^{0}$$

$$\frac{(-3)^4}{4} - \frac{(-2)^4}{4} + \left(\frac{-3(0)^2}{2} + 2(0) - \left(\frac{-3(-2)^2}{2} + 2(-2)\right)\right)$$

$$\frac{81}{4} - \frac{16}{4} + \left(-\left(-6-4\right)\right)$$

$$\frac{81}{4} - \frac{16}{4} + 10 = \frac{81}{4} - \frac{16}{4} + \frac{40}{4} = \frac{105}{4}$$

(b) $\int_{-3}^{5} f(x)\,dx$

part (a)

$$\int_{-3}^{0} f(x)\,dx + \int_{0}^{\pi} (-3x+2)\,dx + \int_{\pi}^{5} \sin x\,dx$$

$$\frac{105}{4} + \frac{-3x^2}{2} + 2x\Big|_{0}^{\pi} + -\cos x\Big|_{\pi}^{5}$$

$$\frac{105}{4} + \frac{-3(\pi)^2}{2} + 2\pi + (-\cos 5 - (-\cos \pi))$$

$$\frac{105}{4} - \frac{3\pi^2}{2} + 2\pi - \cos 5 + \cos \pi$$

$$\frac{105}{4} - \frac{3\pi^2}{2} + 2\pi - \cos 5 - 1$$

$$\frac{101}{4} - \frac{3\pi^2}{2} + 2\pi - \cos 5$$

5. Evaluate $\int_{-3}^{1} h(x)dx$ if $h(x) = \begin{cases} 0 & , x \le 0 \\ 5e^{-5x} & , x > 0 \end{cases}$

$$\int_{-3}^{0} 0\,dx + \int_{0}^{1} 5e^{-5x}dx$$

$$0 + \cancel{5} \cdot -\frac{1}{\cancel{5}} e^{-5x} \Big|_{0}^{1}$$

$$0 + \left(-e^{-5x} \Big|_{0}^{1} \right)$$

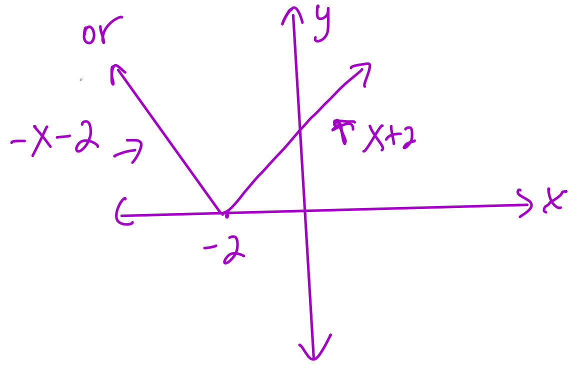
$$0 + \left(-e^{-5} - \left(-e^{0} \right) \right)$$

$$0 + \left(-e^{-5} - (-1) \right)$$

$$0 - e^{-5} + 1$$

$$-e^{-5} + 1 \text{ or } -\frac{1}{e^{5}} + 1$$

7. Evaluate $\int_{-3}^{0} h(x)dx$ if $h(x)=|x+2|$



$$h(x)=\begin{cases} x+2 & x+2\geq 0,\ x\geq -2 \\ -(x+2) & x<-2\end{cases}$$

$$\int_{-3}^{-2} -(x+2)dx + \int_{-2}^{0}(x+2)dx$$

$$\int_{-2}^{-3}(x+2)dx + \int_{-2}^{0}(x+2)dx$$

$$\left.\frac{x^2}{2}+2x\right|_{-2}^{-3} + \left.\frac{x^2}{2}+2x\right|_{-2}^{0}$$

$$\frac{9}{2}-6-\left(\frac{4}{2}-4\right)+\left(0-\left(\frac{4}{2}+2(-2)\right)\right)$$

$$\frac{9}{2}-6+2-0-2+4=\frac{9}{2}-2=\frac{5}{2}$$

Classwork 02-09

(7) $\int_0^1 (2t-1)^3 dt$

$u = 2t-1$
$du = 2dt$
$\frac{du}{2} = dt$

$$\frac{1}{2}\int_{-1}^{1} u^3 du = \frac{1}{2}\cdot\frac{u^4}{4}\Big|_{-1}^{1} = \frac{1}{8}u^4\Big|_{-1}^{1} = \frac{1}{8}\left(1^4 - (-1)^4\right)$$

$$\frac{1}{8}(1-1) = 0 \qquad (D)$$

(8) $\int_4^9 \frac{2+x}{2\sqrt{x}}\,dx = \frac{1}{2}\int_4^9 \left(2x^{-\frac{1}{2}} + x^{\frac{1}{2}}\right)dx$

$$\frac{1}{2}\left(2\cdot 2x^{\frac{1}{2}} + \frac{2}{3}x^{3/2}\right)\Big|_4^9$$

$$\frac{1}{2}\left(4x^{\frac{1}{2}} + \frac{2}{3}x^{3/2}\right)\Big|_4^9$$

$$2x^{\frac{1}{2}} + \frac{1}{3}x^{3/2}\Big|_4^9$$

$$2(9)^{\frac{1}{2}} + \frac{1}{3}(9)^{3/2} - \left(2(4)^{\frac{1}{2}} + \frac{1}{3}(4)^{3/2}\right)$$

$$6 + 9 - \left(4 + \frac{8}{3}\right)$$

$$6+9-4-\frac{8}{3} = 11 - \frac{8}{3} = \frac{33}{3} - \frac{8}{3} = \frac{25}{3}$$

⑨ $\int_0^1 e^{-x}dx$

$u=-x$ UB : -1

$du=-dx$ LB 0

$-du=dx$

$$-\int_0^{-1} e^u du = -e^u \Big|_0^{-1} = -e^{-1}-(-e^0)$$

$$-\frac{1}{e}+1$$

⑩ $\int_0^1 xe^{x^2}dx$

$u=x^2$ UB: 1

$du=2x\,dx$ LB: 0

$du/2=x\,dx$

$$\frac{1}{2}\int_0^1 e^u du \quad \frac{1}{2}e^u\Big|_0^1 \quad \frac{1}{2}(e^1-e^0)=\frac{1}{2}(e-1)$$

⑪ $\int_0^{\pi/4}\sin 2\theta\, d\theta$

$u=2\theta$ UB: $2(\pi/4)=\pi/2$

$du=2d\theta$ LB $2(0)=0$

$\frac{du}{2}=d\theta$

$$\frac{1}{2}\int_0^{\pi/2}\sin u \quad -\frac{1}{2}\cos u\Big|_0^{\pi/2} \quad -\frac{1}{2}(\cos \pi/2-\cos 0)$$

$$-\frac{1}{2}(0-1)$$

$$-\frac{1}{2}(-1)=\frac{1}{2}$$

(12) $\displaystyle\int_1^2 \frac{dz}{3-z}$

$u = 3-z$

$du = -dz$

$-du = dz$

$\displaystyle -\int_2^1 u^{-1}\,du \quad = -\ln|u| \Big|_2^1$

$-\ln 1 - (-\ln 2)$

$0 + \ln 2 = \ln 2$ (E)

Additivity of the Integral

I

① $\int_1^3 f(x)\,dx = \int_1^5 f(x)\,dx - \int_3^5 f(x)\,dx = 3-1 = 2.$

② $\int_2^3 f(x)\,dx + \int^5 f(x)\,dx = 1+1 = 2.$

③ $\int_2^7 f(x)\,dx = \int_2^3 f(x)\,dx + \int_3^7 f(x)\,dx = 1+6 = 7.$

④ $\int_5^7 f(x)\,dx = \int_3^7 f(x)\,dx - \int_3^5 f(x)\,dx = 6-1 = 5.$

⑤ $\int_1^2 f(x)\,dx = \int_1^5 f(x)\,dx - \int_2^3 f(x)\,dx - \int_3^5 f(x)\,dx = 3-1-1 = 1.$

⑥ $\int_1^7 f(x)\,dx = \int_1^5 f(x)\,dx + \int_3^7 f(x)\,dx - \int_3^5 f(x)\,dx = 3+6-1 = 8.$

Because the variable of integration in a definite integral plays no role in the end result, it is often referred to as a dummy variable.

So... whenever you find it convenient to change the letter used for the variable of integration in a definite integral, you can do so w/o changing the value of the integral.

II

⑦ $\int_1^2 g(u)\,du = \int_1^4 g(x)\,dx - \int_2^4 g(r)\,dr = 1-2 = -1$

⑧ $\int_2^9 g(v)\,dv = \int_2^6 g(s)\,ds + \int_6^9 g(t) = 0+1 = 1$

⑨ $\int_4^6 g(w)\,dw = \int_2^6 g(s)\,ds - \int_2^4 g(r)\,dr = 0-2 = -2$

⑩ $\int_1^9 g(x)\,dx = \int_1^2 g(u)\,du + \int_2^6 g(s)\,ds + \int_6^9 g(t)\,dt = -1+0+1 = 0$

⑪ $\int_4^9 g(y)\,dy = \int_2^6 g(s)\,ds + \int_6^9 g(t)\,dt - \int_2^4 g(r)\,dr = 0+1-2 = -1$

⑫ $\int_1^6 g(z)\,dz = \int_1^2 g(u)\,du + \int_2^6 g(s)\,ds = -1+0 = -1$

⑬ $\int_6^4 g(t)\,dt = -\int_4^6 g(t)\,dt = -(-2) = 2$ (done in #9)