Name: ______________________________ Date: ____________

AP Calculus AB: Additional Algebraic Techniques for Anti-differentiation

Do Now:

Evaluate each of the following.

1. $\lim\limits_{x\to 0}\dfrac{e^x - x - 1}{x^2} = \lim\limits_{x\to 0}\dfrac{e^x-1}{2x} = \lim\limits_{x\to 0}\dfrac{e^x}{2} = \dfrac{1}{2}$

2. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}} = \lim\limits_{x\to \frac{\pi}{3}}\dfrac{-\sin x}{1} = -\dfrac{\sqrt{3}}{2}$

3. $\lim\limits_{x\to 0}\dfrac{x}{e^x} = \dfrac{0}{1} = 0$

plug

4. $\frac{\infty}{\infty}$ $\lim\limits_{x\to \infty}\dfrac{e^x}{x^2} = \lim\limits_{x\to \infty}\dfrac{e^x}{2x} = \lim\limits_{x\to \infty}\dfrac{e^x}{2} = \infty$

5. $\frac{0}{0}$ $\lim\limits_{x\to \pi}\dfrac{1+\cos x}{\sin 2x} = \lim\limits_{x\to \pi}\dfrac{-\sin x}{2\cos 2x} = \dfrac{0}{2} = 0$

Additional Algebraic Techniques for Anti-differentiation

1. $\displaystyle\int \frac{x^2}{x+3}\,dx$

Synthetic Division

$$\begin{array}{r|rrr} -3 & 1 & 0 & 0 \\ & & -3 & 9 \\ \hline & 1 & -3 & 9 \end{array}$$

$x - 3 + \dfrac{9}{x+3}$

Long Division

$$\begin{array}{r} x-3 \\ x+3\,\overline{)\,x^2} \\ \underline{x^2+3x} \\ -3x \\ \underline{-3x-9} \\ 9 \end{array}$$

$x - 3 + \dfrac{9}{x+3}$

$\displaystyle\int \left(x - 3 + \frac{9}{x+3}\right)dx$

$\displaystyle\int (x-3)\,dx + \int \frac{9}{x+3}\,dx$

$\displaystyle\frac{x^2}{2} - 3x + 9\int \frac{1}{x+3}\,dx$ $\qquad u = x+3$, $du = dx$

$\displaystyle\frac{x^2}{2} - 3x + 9\int \frac{1}{u}\,du$

$\dfrac{x^2}{2} - 3x + 9\ln|u| + C$

$\dfrac{x^2}{2} - 3x + 9\ln|x+3| + C$

2. $\displaystyle\int_{-2}^{1} \frac{x^2}{x+3}\,dx$

↑ same integral

$\dfrac{1^2}{2} - 3(1) + 9\ln|1+3| - \left(\dfrac{(-2)^2}{2} - 3(-2) + 9\ln|-2+3|\right)$ $\quad 0$

$\dfrac{1}{2} - 3 + 9\ln 4 - (2 + 6)$

$\dfrac{1}{2} - 3 + 9\ln 4 - 8$

$\dfrac{1}{2} - 11 + 9\ln 4$

$-\dfrac{21}{2} + 9\ln 4$

completing the square

$x^2-6x+13$

$\underbrace{x^2-6x+9}-9+13$

$(x-3)^2+4$

3. $\displaystyle\int \frac{1}{2x^2-12x+26}\,dx$

$\frac{1}{2}\int \frac{1}{x^2-6x+13}\,dx$

goal: $\frac{1}{u^2+1}$

$\frac{1}{2}\int \frac{1}{(x-3)^2+4}\,dx$

$\frac{1}{2}\int \frac{1}{4\left(\frac{(x-3)^2}{4}+1\right)}$

$\frac{1}{2}\cdot\frac{1}{4}\int \frac{1}{\left(\frac{x-3}{2}\right)^2+1}\,dx$

$u=\frac{x-3}{2}$ or $\frac{1}{2}(x-3)$

$du=\frac{1}{2}dx$

$2du=dx$

$\frac{1}{8}\cdot 2\int \frac{1}{u^2+1}\,du$

$\frac{1}{4}\arctan u+C$

$\frac{1}{4}\arctan\left(\frac{x-3}{2}\right)+C$

4. $\displaystyle\int \frac{dx}{\sqrt{21-4x-x^2}}$

goal $\frac{1}{\sqrt{1-u^2}}$

$-x^2-4x+21$

$-(x^2+4x+4-4-21)$ ($-4-21=-25$) $\quad -(x+2)^2-(-25)$

$-(x+2)^2+25$

$25-(x+2)^2$

$\int \frac{dx}{\sqrt{25-(x+2)^2}}$

$\int \frac{dx}{\sqrt{25\left(1-\frac{(x+2)^2}{25}\right)}}$

$\int \frac{dx}{\sqrt{25}\sqrt{1-\frac{(x+2)^2}{25}}}$

$\frac{1}{5}\int \frac{dx}{\sqrt{1-\left(\frac{x+2}{5}\right)^2}}$

$u=\frac{x+2}{5}$

$du=\frac{1}{5}dx$

$5du=dx$

$\cancel{\frac{1}{5}}\cdot\cancel{5}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\sin^{-1}u+C$

$\sin^{-1}\left(\frac{x+2}{5}\right)+C$

Homework 02-14

Name: Key

Date: ____________

AP Calculus AB: Evaluating Definite Integrals of Piecewise Functions

Ms. Loughran

1. Evaluate $\int_0^8 h(x)dx$ if $h(x)=\begin{cases}4 & ,x<4\\ x & ,x\geq 4\end{cases}$

$\int_0^4 4dx+\int_4^8 xdx$

$4x\Big|_0^4 + \frac{x^2}{2}\Big|_4^8 = 16-0+(32-8) = 40$

2. Evaluate $\int_0^4 g(x)dx$ if $g(x)=\begin{cases}2x-5 & ,x<2\\ x^2 & ,x\geq 2\end{cases}$

$\int_0^2 (2x-5)dx + \int_2^4 x^2dx$

$x^2-5x\Big|_0^2 + \frac{x^3}{3}\Big|_2^4 = -6+\left(\frac{64}{3}-\frac{8}{3}\right) = \frac{38}{3}$

3. Evaluate $\int_{-3}^0 f(x)dx$ if $f(x)=\begin{cases}-x^3 & ,x<-2\\ -3x+2 & ,-2\leq x<\pi\\ \sin x & ,x\geq\pi\end{cases}$

$\int_{-3}^{-2} -x^3dx + \int_{-2}^0 (-3x+2)dx = -\frac{x^4}{4}\Big|_{-3}^{-2} + -\frac{3x^2}{2}+2x\Big|_{-2}^0$

$-\frac{16}{4}-\frac{81}{4}$

$\frac{65}{4}+0-(-6-4)=\frac{105}{4}$

3b) $\int_{-3}^{-2} -x^3dx+\int_{-2}^{\pi}(-3x+2)dx+\int_{\pi}^{5}\sin x\,dx = \frac{65}{4}+\frac{-3\pi^2}{2}+2\pi+10-\cos 5-1$

$=\frac{101}{4}-\frac{3\pi^2}{2}+2\pi-\cos 5$

4. Evaluate $\int_{-2}^5 f(x)dx$ if $f(x)=\begin{cases}-2x^2 & ,x<0\\ 5x & ,0\leq x<2\\ \frac{1}{x} & ,x\geq 2\end{cases}$

$\int_{-2}^0 -2x^2dx+\int_0^2 5xdx+\int_2^5 \frac{1}{x}dx = -\frac{2x^3}{3}\Big|_{-2}^0 + \frac{5x^2}{2}\Big|_0^2 + \ln|x|\Big|_2^5 = -\frac{16}{3}+10+\ln 5-\ln 2$

$\frac{14}{3}+\ln\frac{5}{2}$

5. Evaluate $\int_{-3}^1 h(x)dx$ if $h(x)=\begin{cases}0 & ,x\leq 0\\ 5e^{-5x} & ,x>0\end{cases}$

$\int_{-3}^0 0\,dx+\int_0^1 5e^{-5x}dx = 5\cdot -\frac{1}{5}e^{-5x}\Big|_0^1$

$-e^{-5x}\Big|_0^1$

$-e^{-5}-(-e^0)$

$\boxed{-e^{-5}+1}$

6. Evaluate $\int_{-2}^4 f(x)dx$ if $f(x)=\begin{cases}0 & ,x<-1\\ 9x & ,-1\leq x<3\\ 3e^x & ,3\leq x\leq 5\\ 0 & ,x>5\end{cases}$

$\int_{-2}^{-1} 0dx+\int_{-1}^3 9xdx+\int_3^4 3e^xdx$

$0+\frac{9x^2}{2}\Big|_{-1}^3+3e^x\Big|_3^4 = \boxed{0+36+3e^4-3e^3}$