Do Now:

2003 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.
   (a) Find the acceleration of the particle at time $t = 3$.
   (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.
   (c) Find all values of t at which the particle changes direction. Justify your answer.
   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

a) $a(t) = -e^{1-t}$

b) $v(3) = -1 + e^{1-3} = -1 + e^{-2} < 0$

$a(3) = -e^{1-3} = -e^{-2} < 0$

The particle is speeding up b/c $a(3)$ and $v(3)$ are both negative.

c) $v(t) = -1 + e^{1-t}$

$-1 + e^{1-t} = 0$

$e^{1-t} = 1$

$1 - t = 0$

$t = 1$

$v(t)$: $+$ on $(0, 1)$, $-$ after 1

$-1 + e^{1-\frac{1}{2}}$

$-1 + e^{\frac{1}{2}}$

we check $v(3)$ before

The particle changes direction at $t = 1$ b/c velocity changes sign at $t = 1$.

d) $\int_0^3 |v(t)|\,dt = \int_0^1 v(t)\,dt + \int_3^1 v(t)\,dt$ OR

$X(0) = -e^1 + C$

$X(1) = -1 - e^{1-1} = -2 + c$

$X(3) = -3 - e^{-2} + c$

$X(t) = -t - e^{1-t} + c$

TD: $|-e + 2| + |-2 - (-3 - e^{-2})|$

$x(3) = 3 \quad 6 \cdot 6$

Rectilinear Motion Revisited Packet
For any questions in this packet prior to 2000 do not use the graphing calculator.
(Use of graphing calculator on AP exam began in 1995.)
Then follow these guidelines:
2000-2010 #s 1-3 are calculator active and #s 4-6 are non-calculator
2011- present #s 1-2 are calculator active and 3-6 are non-calculator

1987 AB1

A particle moves along the x-axis so that its acceleration at any time t is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

(a) Write an expression for the velocity $v(t)$ of the particle at any time t.

(b) For what values of t is the particle at rest?

(c) Write an expression for the position $x(t)$ of the particle at any time t.

(d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

$v(0) = 24$

a) $\int (6t-18)\,dt = 3t^2 - 18t + C$

$24 = 3(0)^2 - 18(0) + c$

$24 = c$

$v(t) = 3t^2 - 18t + 24$

b) $0 = 3t^2 - 18t + 24$

$0 = 3(t^2 - 6t + 8)$

$0 = 3(t-4)(t-2)$

$t = 4, 2$

c) $\int (3t^2 - 18t + 24)\,dt$

$x(t) = t^3 - 9t^2 + 24t + C$

$20 = 1 - 9 + 24 + c$

$20 = 16 + c$

$c = 4$

$x(t) = t^3 - 9t^2 + 24t + 4$

d) $x(1) = 20 \quad \} \; 4$

$x(2) = 24 \quad \} \; 2$

$x(3) = 22$

$TD = 6$

2006 AB4

t (seconds)	$\cancel{0}$	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	$\cancel{5}$	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

$\frac{v(80)-v(0)}{80} = \frac{49-5}{80}$ ft/s²

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

$\int_{10}^{70} v(t)\,dt$ represents the rocket's change in position from 10 to 70 s. Since $v(t) > 0$ from 10 to 70, the integral represents to total distance travelled from 10 to 70

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

$$(20)(22) + (20)(35) + (20)(44) \text{ ft}$$

c) $$\int \frac{3}{\sqrt{t+1}}\,dt = 3\int \frac{1}{\sqrt{t+1}}\,dt = 3\int u^{-\frac{1}{2}}\,du$$

$u = t+1$
$du = dt$

$$= 3 \cdot 2u^{\frac{1}{2}} + C$$

$$v(t) = 6\sqrt{t+1} + C$$

$$2 = 6\sqrt{0+1} + C$$

$$-4 = C$$

$$V_B(t) = 6\sqrt{t+1} - 4$$

$$V_B(80) = 6\sqrt{80+1} - 4 = 50 \text{ ft/s}$$

Rocket B

Homework 02-23

AB Calc Even More Rectilinear Motion Problems

1989 AB 3

$a(t) = 4\cos(2t)$

$v(0) = 1$

(a) $\int a(t) = \int 4\cos(2t) = 4\int \cos(2t) = 4\cdot\frac{1}{2}\int \cos u = 2\sin u + C$

$u = 2t$ $\qquad v(t) = 2\sin(2t) + C$

$du = 2\,dt$

$v(0) = 1$

$1 = 2\sin(2(0)) + C$

$1 = 0 + C$

$1 = C$

$v(t) = 2\sin(2t) + 1$ $\qquad \int \sin u = -\cos u$

(b) $x(t) = \int v(t) = \int 2\sin 2t + 1 = 2\int \sin 2t\,dt + \int 1\,dt$

$u = 2t$

$du = 2dt$

$x(t) = -\cos 2t + t + C$

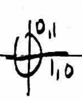
$x(0) = 0 \qquad 0 = -\cos 2(0) + 0 + C$

$0 = -1 + C$

$1 = C$

$x(t) = -\cos 2t + t + 1$

(c) $v(t) = 0 \qquad 2\sin(2t) + 1 = 0$



$2\sin(2t) = -1$

$\sin(2t) = -\frac{1}{2}$

$2t = \frac{7\pi}{6}, \frac{11\pi}{6} \qquad t = \frac{7\pi}{12}, \frac{11\pi}{12}$

1990 AB 1

$a(t) = 12t^2 - 4$ $\qquad x(1) = 3$ $\qquad$ initially at rest $v(0) = 0$

(a) $v(t) = \int a(t) = \int 12t^2 - 4 \quad = \quad 4t^3 - 4t + c$ $\qquad 4t^3 - 4t = 0$

$v(0) = 0$ $\qquad 4t^3 - 4t + 0$ $\qquad 4t(t^2 - 1) = 0$

$v(t) = 4t^3 - 4t$ $\qquad t = 0 \mid t = \pm 1$

reject -1

@ $t = 0, t = 1$

(b) $x(t) = \int v(t) = \int 4t^3 - 4t \quad = \quad t^4 - 2t^2 + c$

$x(1) = 3$

$3 = 1^4 - 2(1)^2 + c$

$3 = 1 - 2 + c$ $\qquad x(t) = t^4 - 2t^2 + 4$

$3 = -1 + c$

$4 = c$

(c) total distance $= \int_0^2 |v(t)|\,dt \quad = \int_0^2 4t^3 - 4t \quad = 10$

$-$ $+$ $v(t)$

0 1 2

OR

$-\int_0^1 4t^3 - 4t + \int_1^2 4t^3 - 4t$

$x(0) = 4$

$x(1) = 3$ $\quad 1$ $\qquad \int_1^0 4t^3 - 4t$

$x(2) = 12$ $\quad 9$ $\qquad \frac{4t^4}{4} - \frac{4t^2}{2}$

$\overline{10}$ $\qquad t^4 - 2t^2 \Big|_1^0$

$8 - (-1)$

$16 - 8$

$0^4 - 2(0)^2 - (1^4 - 2(1)^2) + [2^4 - 2(2)^2 - (1^4 - 2(1)^2)]$

$0 + 1 + 9$

$1 + 9 = 10$

2002 AB 3

$v(t) = \sin\left(\frac{\pi}{3}t\right)$

(a) $a(t) = \cos(\pi/3\, t) \cdot \pi/3$

$a(t) = \pi/3 \cos(\pi/3\, t)$

$a(4) = \pi/3 \cos(\pi/3 \cdot 4)$

$a(4) = \pi/3 \cos(4\pi/3)$

$a(4) = \pi/3 \left(-\frac{1}{2}\right) = -\pi/6 = -0.523$

(b) at $t=4$ acc $\ominus$

vel $\ominus$

speed ↑

Both are correct: I is True $a(t)<0$

II is True, $v(t)<0$ and $a(t)<0$

(c) $\int_0^4 |v(t)| = 2.387$

$x(4) = x(0) + \int_0^4 v(t)\,dt$

displacement

(d) $x(t) = \int \sin(\pi/3\, t)\,dt$

$u = \pi/3\, t$

$\frac{du}{\pi/3} = \frac{\pi/3\,dt}{\pi/3}$

$x(t) = 3/\pi \cdot -\cos \pi/3\, t + C$

$x(t) = -3/\pi \cos \pi/3\, t + C$

$x(0) = 2$

$2 = -3/\pi \cos \pi/3(0) + C$

$2 = (-3/\pi)(1) + C$

$2 = -3/\pi + C$

$C = 2 + 3/\pi$

ON CAC

$2 + \frac{9}{2\pi}$

$x(t) = -3/\pi \cos \pi/3\, t + 2 + 3/\pi$

$x(4) = -3/\pi \cos \pi/3(4) + 2 + 3/\pi$

$= -3/\pi \cos 4\pi/3 + 2 + 3/\pi$

$= -3/\pi\left(-\frac{1}{2}\right) + 2 + 3/\pi$

$= 3/2\pi + (3/\pi) + 2$

$= 2 + 9/2\pi = 3.432$

2003 AB 2

$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right) = y_1$ $\qquad y_2 = \text{nDeriv}(Y_1, X, X)$ ← acceleration

(a) $a(2) = 1.588$

Since $a(2) > 0$ and $v(2) <$ the speed is decreasing at $t = 2$.

(b) $v(t) = 0$, $0 < t < 3$

$0 = -(t+1)\sin\left(\frac{t^2}{2}\right)$

can never $= 0$

$t \neq -1$

$\sin\left(\frac{t^2}{2}\right) = 0$

$t^2/2 = \pi$

$t^2 = 2\pi$

$t = \sqrt{2\pi}$

or

(0,1) (1,0) (1,0)

on calc $2.5066 \approx 2.506$

(c) $\int_0^3 |v(t)| = 4.333818$ $\qquad 4.334$

(d) $X(0) = 1$ displacement FTC

$\int_0^{\sqrt{2\pi}} v(t)\,dt = s(\sqrt{2\pi}) - s(0)$ given

$-3.2654 = s(\sqrt{2\pi}) - 1$

$-2.2654 = s(\sqrt{2\pi})$

$\int_{\sqrt{2\pi}}^3 v(t)\,dt = s(3) - s(\sqrt{2\pi})$

$1.0683 = s(3) - (-2.265)$

$1.0683 = s(3) + 2.265$

$-1.197 = s(3)$

t	$s(t)$
0	1 given
$\sqrt{2\pi}$	-2.265
3	-1.197

∴ the greatest distance from the origin is 2.265