Name: ______________________________ Date: ______________

AP Calculus AB Ms. Loughran

Do Now:

1. $\int_{\pi}^{2\pi} \sin x dx =$ $-\cos x \Big|_{\pi}^{2\pi} = -(\cos 2\pi - \cos \pi) = -2$ $\quad 1-(-1)$

2. $\int_{0}^{\frac{\pi}{4}} \sec^2 x dx =$ $\tan x \Big|_{0}^{\frac{\pi}{4}} = \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

3. $\int_{-2}^{6} 5dx =$ $5x \Big|_{-2}^{6} = 30 - (-10) = 40$

4. $\int_{-1}^{1} \frac{dx}{1+x^2} =$ $\arctan x \Big|_{-1}^{1} = \arctan 1 - \arctan(-1)$

$\frac{\pi}{4} - \left(\frac{-\pi}{4}\right) = \frac{\pi}{2}$

5. $\int_{0}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} =$ $\sin^{-1}(x) \Big|_{0}^{\frac{1}{2}} = \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$\frac{\pi}{6} - 0 = \frac{\pi}{6}$

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AP Calc AB Ms. Loughran

1987 AB 1

A particle moves along the x-axis so that its acceleration at any time t is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$ its position is $x(1) = 20$.

(a) Write an expression for the velocity $v(t)$ of the particle at any time t.

(b) For what values of t is the particle at rest?

(c) Write an expression for the position $x(t)$ of the particle at any time t.

(d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

(a) $v(t) = \int (6t-18)\,dt = 3t^2 - 18t + C$ $\quad v(0) = 24$

$24 = 3(0)^2 - 18(0) + C$

$24 = C$

$v(t) = 3t^2 - 18t + 24$

(b) $3t^2 - 18t + 24 = 0$

$t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

$t = 2, 4$

(c) $x(t) = \int (3t^2 - 18t + 24)\,dt$ $\quad x(1) = 20$

$x(t) = t^3 - 9t^2 + 24t + C$

$20 = 1^3 - 9(1)^2 + 24(1) + C$

$20 = 16 + C$

$4 = C$

$x(t) = t^3 - 9t^2 + 24t + 4$

(d) $x(1) = 20$

$x(2) = 8 - 36 + 48 + 4 = 24$ $\quad 4$

$x(3) = 27 - 81 + 72 + 4 = 22$ $\quad 2$

TD: 6

Differential Equations with Initial Conditions

1. State an equation of the curve whose slope at the point (x, y) is $3x^2$ if the curve contains the point whose coordinates are (1,-1).

$$\frac{dy}{dx} = 3x^2$$

$$\int dy = \int 3x^2\,dx$$

$$y = x^3 + c \qquad (1,-1)$$

$$-1 = (1)^3 + c$$

$$-2 = c$$

$$y = x^3 - 2$$

3. Find f(x) if $f''(x) = x^{-\frac{3}{2}}, f'(4) = 2$ and f(0)=0.

$$f'(x) = \int x^{-3/2}\,dx = -2x^{-\frac{1}{2}} + c \qquad f'(4) = 2$$

$$2 = -2(4)^{-\frac{1}{2}} + c$$

$$2 = -1 + c$$

$$3 = c$$

$$f'(x) = -2x^{-\frac{1}{2}} + 3$$

$$f(x) = \int \left(-2x^{-\frac{1}{2}} + 3\right)dx \qquad f(0) = 0$$

$$f(x) = -4x^{\frac{1}{2}} + 3x + c$$

$$0 = c$$

$$f(x) = -4\sqrt{x} + 3x$$

4. $f''(x) = 6(x-1)$. Find f(x) if f(2) = 1 and at the point whose coordinates are (2,1), the graph of y = f(x) is tangent to the line given by the equation 3x-y-5=0.

$3x-5=y$

$m=3$

$f'(2)=3$

$$f'(x) = \int (6x-6)\,dx = 3x^2-6x+c$$

$$3 = 3(2)^2-6(2)+c$$

$$3 = c$$

$$f'(x) = 3x^2-6x+3$$

$f(2)=1$

$$f(x) = \int (3x^2-6x+3)\,dx = x^3-3x^2+3x+c$$

$$1 = 2^3-3(2)^2+3(2)+c$$

$$1 = 2+c$$

$$-1 = c$$

$$f(x) = x^3-3x^2+3x-1$$

$X(0) = 0$

2. Acceleration due to gravity is -32 feet per second per second. A stone is thrown upward from the ground with an initial speed of 96 feet per second.

$v(0) = 96$

(A) Find the height to which the stone rises in t seconds.
(B) Find its maximum height.
(C) When is the velocity of the stone one-half its initial velocity?
(D) What is the height of the stone when its velocity is one-half its initial velocity?

(a) $v(t) = \int -32\,dt = -32t + C$

$96 = -32(0) + C$

$96 = C$

$v(t) = -32t + 96$

$x(t) = \int (-32t + 96)\,dt = -16t^2 + 96t + C$ $\quad x(0) = 0$

$0 = C$

$x(t) = -16t^2 + 96t$

(B) $v(t) = -32t + 96 = 0$

$t = 3$ sec

$x(3) = -16(3)^2 + 96(3)$ ft

(C) $48 = v(t)$

$48 = -32t + 96$

$-48 = -32t$

$\frac{48}{32} = t$

$t = 3/2$ sec.

(D) $x(\frac{3}{2}) = -16(\frac{3}{2})^2 + 96(\frac{3}{2})$ ft

Homework 02-26

Name: Ky   Date: ____________
AP Calculus AB: Separable Differential Equations Homework   Ms. Loughran

For each of the following, solve for y.

1. Solve for y if $\frac{dy}{dx} = -\sin x$ and $y = 2$ when $x = 0$.

$\int dy = \int -\sin x \, dx$

$y = \cos x + C$

$(0,2)$

$2 = \cos 0 + C$

$2 = 1 + C$

$C = 1$

$y = \cos x + 1$

2. $\frac{dy}{dx} = \sqrt[3]{x}$ and $y = 2$ when $x = 1$.

$\int dy = \int x^{\frac{1}{3}} dx$

$y = \frac{3}{4} x^{\frac{4}{3}} + C$

$(1,2)$

$2 = \frac{3}{4}(1)^{\frac{4}{3}} + C$

$2 = \frac{3}{4} + C$

$\frac{8}{4} - \frac{3}{4} = C$

$\frac{5}{4} = C$

$y = \frac{3}{4} x^{\frac{4}{3}} + \frac{5}{4}$

3. $\frac{dy}{dx} = \frac{1}{x}$ and $y = 5$ when $x = -1$.

$x\,dy = dx$

$\int dy = \int \frac{dx}{x}$

$\int dy = \int x^{-1} dx$

$y = \ln|x| + C$

$(-1, 5)$

$5 = \ln|-1| + C$

$5 = \ln 1 + C$

$\ln 1 = 0$

$5 = 0 + C$

$C = 5$

$y = \ln|x| + 5$

4. $\frac{dy}{dx} = \frac{x+1}{\sqrt{x}}$ and $y = 0$ when $x = 1$.

$\sqrt{x}\,dy = x + 1\,dx$

$\int dy = \int \frac{x+1}{\sqrt{x}} dx$

$y = \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right) dx$

$(1, 0)$

$y = \frac{2}{3}x^{3/2} + 2x^{\frac{1}{2}} + C$

$0 = \frac{2}{3}(1)^{3/2} + 2(1)^{\frac{1}{2}} + C$

$0 = \frac{2}{3} + 2 + C$

$0 = \frac{8}{3} + C$

$C = -8/3$

$y = \frac{2}{3}x^{3/2} + 2x^{\frac{1}{2}} - \frac{8}{3}$

5. Find the general form of a function, $(f(x))$, whose second derivative is $f''(x) = \sqrt{x}$.

$f''(x) = \sqrt{x}$

$f'(x) = \int x^{\frac{1}{2}} dx$

$f'(x) = \frac{2}{3} x^{3/2} + C$

$f(x) = \int \frac{2}{3x}^{3/2} + C \; dx$

$\frac{2}{3} \cdot \frac{2}{5} x^{5/2} + Cx + C_2$

$\frac{4}{15} x^{5/2} + C x + C_2$