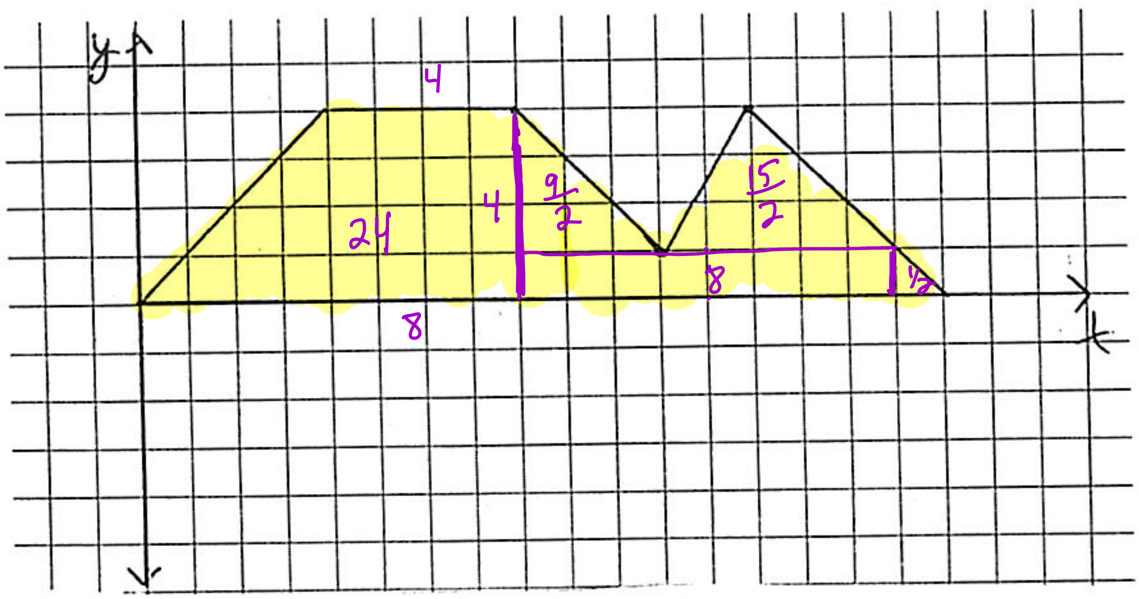
Name: ____ Date: ____________

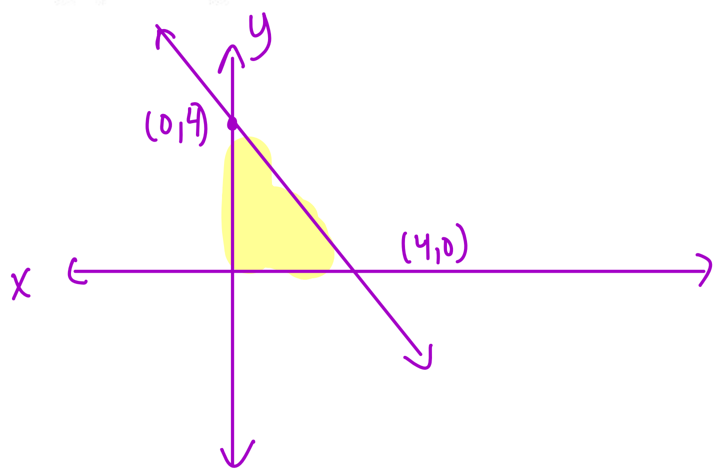
AP Calc AB: Introduction to Area Under a Curve

1. Find the area between the function and the x-axis (otherwise known as the area under the curve.)



$A = 44.5$

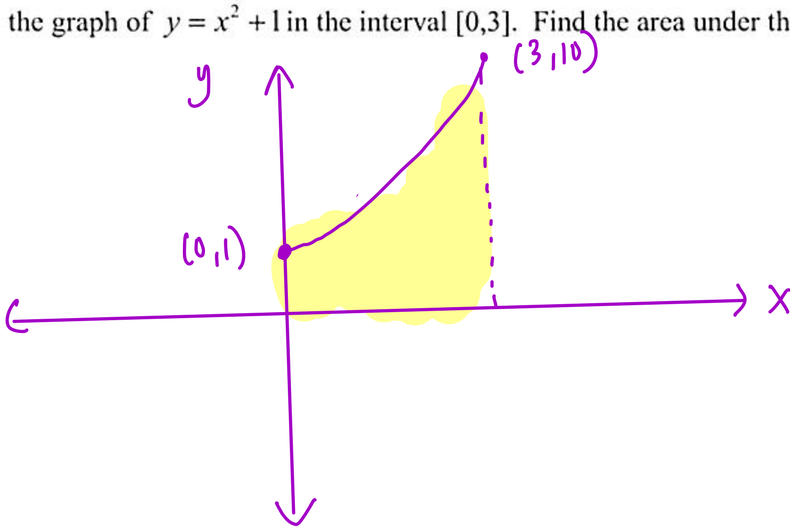
2. Sketch the graph of $x + y = 4$. Find the area under the curve.



$y = -x + 4$

$A = \frac{4(4)}{2} = 8$

3. Sketch the graph of $y = x^2 + 1$ in the interval [0,3]. Find the area under the curve.



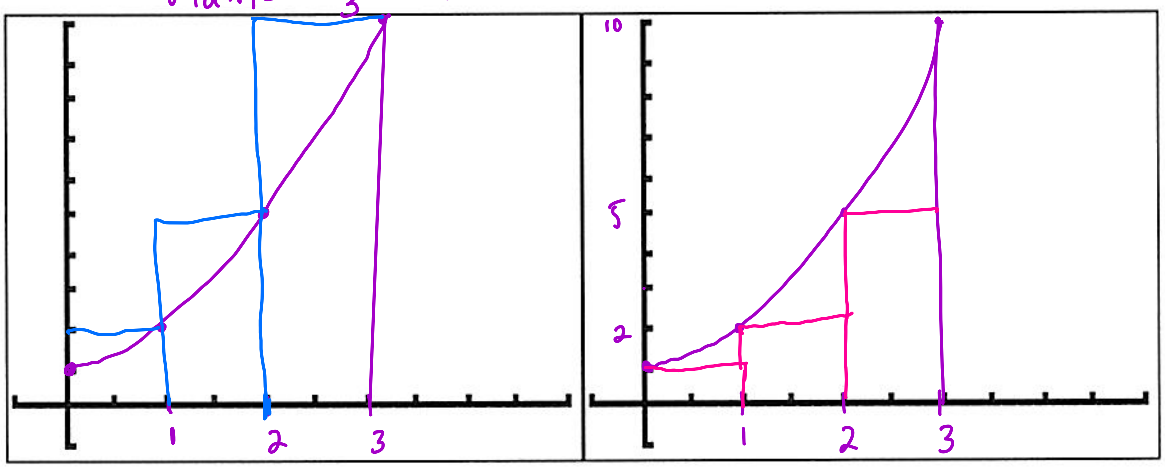
We cannot find the area of this region using any formula we know since this is not a shape we have an area formula for. HOWEVER, we can make rectangles under this curve since we know the area of a rectangle.

How? By drawing in rectangles with equal widths.

→ 3 rectangles

Let's start by using 3 equal subintervals. There are two ways we can approach this.

$y = x^2 + 1$ $\quad$ width $= \frac{3-0}{3} = 1$



RIGHT ENDPOINT
To draw these correctly, the right endpoint of each rectangle must be on the graph.

LEFT ENDPOINT
To draw these correctly, the left endpoint of each rectangle must be on the graph.

Let's let 2 boxes represent one unit on the x axis.

$A = 1(2) + 1(5) + 1(10)$

$A = 1(2+5+10)$

$A = 17$

↑ over approximation

$A = 1(1) + 1(2) + 1(5)$

$A = 1(1+2+5)$

$A = 8$

↑ under approximation

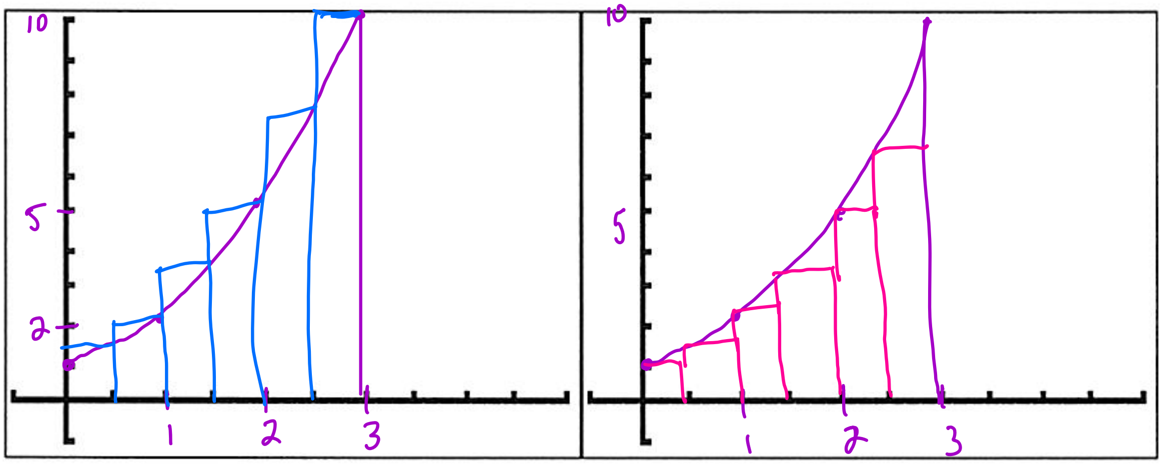
x	y
0	1
.5	1.25
1	2
1.5	3.25
2	5
2.5	7.25
3	10

Using the same example, let's calculate the area if we made 6 rectangles.

RIGHT ENDPOINT

$\text{width} = \frac{3-0}{6} = \frac{1}{2}$

LEFT ENDPOINT



Calculations:

Right Endpoint

$$A = \frac{1}{2}(1.25 + 2 + 3.25 + 5 + 7.25 + 10)$$

$$A = 14.375$$

more accurate but still an over approximation

Left Endpoint

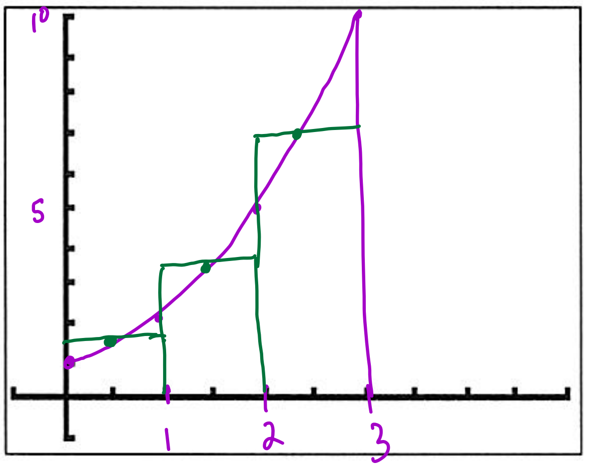
$$A = \frac{1}{2}(1 + 1.25 + 2 + 3.25 + 5 + 7.25)$$

$$A = 9.875$$

more accurate than when we used 3 rectangles but still an underestimate

Midpoint Rule

Let's use the same example but instead of a right or left endpoint approximation, lets use 3 equal subintervals and do a midpoint approximation. The only difference is how we define the heights of the rectangles again.



x	y
0	1
.5	1.25
1	2
1.5	3.25
2	5
2.5	7.25
3	10

$$A = 1(1.25) + 1(3.25) + 1(7.25)$$

$$A = 11.75$$

The process of using rectangles to approximate area under a curve is called **Riemann sums**. A Riemann sum is an approximation of area calculated using rectangles

Homework 02-27

Name: ____________________________ Date: _____________
AP Calculus AB Ms. Loughran

1. If $g(x)$ is continuous for all values of x, then $\int_{\frac{a}{3}}^{\frac{b}{3}} g(3x)\,dx =$

$u = 3x$
$du = 3dx$
$\frac{du}{3} = dx$

$\frac{1}{3}\int_a^b g(u)\,du$

(A) $\frac{1}{3}\int_a^b g(x)\,dx$

(B) $3\int_a^b g(x)\,dx$

(C) $\frac{1}{3}\int_{3a}^{3b} g(x)\,dx$

(D) $\int_a^b g(x)\,dx$

(E) $3\int_{3a}^{3b} g(x)\,dx$

$\int \frac{dy}{y^2} = \int 2dx$
$-\frac{1}{y} = 2x + c \quad (1,-1)$
$-\frac{1}{-1} = 2(1) + c$
$1 = 2 + c$
$-1 = c$

$-\frac{1}{y} = 2x - 1$
when $x = 2$
$-\frac{1}{y} = 2(2) - 1$
$-\frac{1}{y} = 3$
$y = -\frac{1}{3}$

2. If $\frac{dy}{dx} = 2y^2$ and $y = -1$ when $x = 1$ when $x = 2$, $y =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

3. If $\int_k^2 (2x-2)\,dx = -3$ a possible value of k is

$x^2 - 2x \Big|_k^2 = -3$

$2^2 - 2(2) - (k^2 - 2k) = -3$
$0 - k^2 + 2k = -3$
$0 = k^2 - 2k - 3$
$0 = (k-3)(k+1)$
$k = 3 \quad k = -1$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 3

4. $\int_{-1}^{1} \frac{4}{1+x^2}\,dx =$ $4\int_{-1}^{1} \frac{1}{1+x^2}\,dx = 4\arctan x \Big|_{-1}^{1}$ $4\arctan(1) - 4\arctan(-1)$
$4\left(\frac{\pi}{4}\right) - 4\left(-\frac{\pi}{4}\right) = \pi + \pi$

(A) 0 (B) π (C) 1 (D) 2π (E) 2

5. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle changes direction at

$v(0)=10$ $\quad$ $x(0)=4$

$v(t) = \int (4t-12)\,dt = 2t^2 - 12t + c$

$10 = 2(0)^2 - 12(0) + c$

$10 = c$

$v(t) = 2t^2 - 12t + 10$

$2t^2 - 12t + 10 = 0$

$2(t^2 - 6t + 5) = 0$

$2(t-1)(t-5) = 0$

$t = 1, 5$

$v(t)$: $+$ $\quad 1 \quad$ $-$ $\quad 5 \quad$ $+$

(A) $t = 1$
(B) $t = 3$
(C) $t = 5$
(D) $t = 1$ and $t = 5$
(E) $t = 1$, $t = 3$, and $t = 5$

6. If $\int_{30}^{100} f(x)\,dx = A$ and $\int_{50}^{100} f(x)\,dx = B$, then $\int_{30}^{50} f(x)\,dx =$ $\int_{30}^{100} - \int_{50}^{100} = A - B$

$\int_{30}^{50} + \int_{50}^{100} = \int_{30}^{100}$

(A) $A + B$ $\quad$ (B) $A - B$ $\quad$ (C) 0 $\quad$ (D) $B - A$ $\quad$ (E) 20

7. Which of the following integrals correctly gives the area of the region consisting of all points above the x-axis and below the curve $y = 8 + 2x - x^2$?

(A) $\int_{-2}^{4} (x^2 - 2x - 8)\,dx$ $\quad$ (C) $\int_{-2}^{4} (8 + 2x - x^2)\,dx$ $\quad$ (E) $\int_{2}^{4} (8 + 2x - x^2)\,dx$

(B) $\int_{-4}^{2} (8 + 2x - x^2)\,dx$ $\quad$ (D) $\int_{-4}^{2} (x^2 - 2x - 8)\,dx$

$u = \sin^2 x$

$du = 2\sin x \cos x\,dx$

$du = \sin(2x)\,dx$

8. $\int_{0}^{\frac{\pi}{2}} \sin(2x) e^{\sin^2 x}\,dx =$ $\int_0^1 e^u\,du = e^u \Big|_0^1 = e^1 - e^0 = e - 1$

(A) e $\quad$ (B) $e - 1$ $\quad$ (C) $1 - e$ $\quad$ (D) $e + 1$ $\quad$ (E) 1

Calculator Active

9. If the definite integral $\int_{1}^{3} (x^2 + 1)\,dx$ is approximated by using the Trapezoid Rule with $n = 4$, the error is

(A) 0 $\quad$ (B) $\frac{7}{3}$ $\quad$ (C) $\frac{1}{12}$ $\quad$ (D) $\frac{65}{6}$ $\quad$ (E) $\frac{97}{3}$

10. Find the distance traveled in the first four seconds, for a particle whose velocity is given by $v(t) = 7e^{-x^2}$, where t stands for time.

(A) 0.976 $\quad$ (B) 6.204 $\quad$ (C) 6.359 $\quad$ (D) 12.720 $\quad$ (E) 7.000