Do Now:

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

$$\frac{f(5)-f(3)}{5-3} = \frac{-2-4}{2} = -3$$

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

$$1(1)+2(4)+3(-2)+5(3) = 18$$

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

---

b) $$\int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$$

$$3x\Big|_2^{13} - 5\left[f(13) - f(2)\right]$$

$$3(13) - 3(2) - 5[6-1]$$

$$39 - 6 - 25 = 8$$

d) $$y + 2 = 3(x-5)$$

$$y = 3(x-5) - 2$$

$$y = 3(7-5) - 2$$

$$y = 4$$

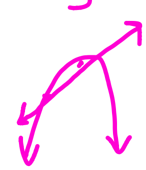
Since $f''(x) < 0$, f is concave down so the tangent line lies above the curve

$$m_{sec} = \frac{f(8)-f(5)}{8-5} = \frac{3-(-2)}{3} = \frac{5}{3}$$

$$y + 2 = \frac{5}{3}(x-5)$$

$$y = \frac{5}{3}(x-5) - 2$$

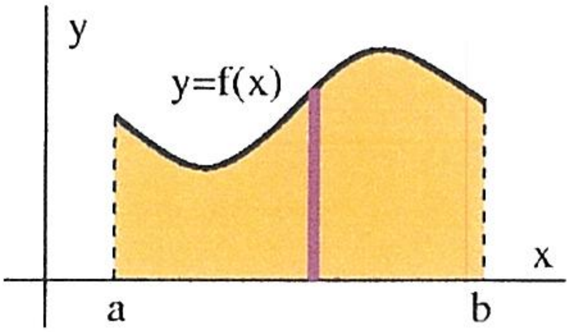
$$y = \frac{5}{3}(7-5) - 2 = \frac{4}{3}$$



Name: ______________________________ Date: ___________

AP Calculus AB: Area Between 2 Curves Ms. Loughran

Remember:



If a function f is continuous on $[a,b]$ and if $f(x) \geq 0$ for all x in $[a,b]$ then the area under the curve $y = f(x)$ over the interval $[a,b]$ is defined by:

$$Area = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

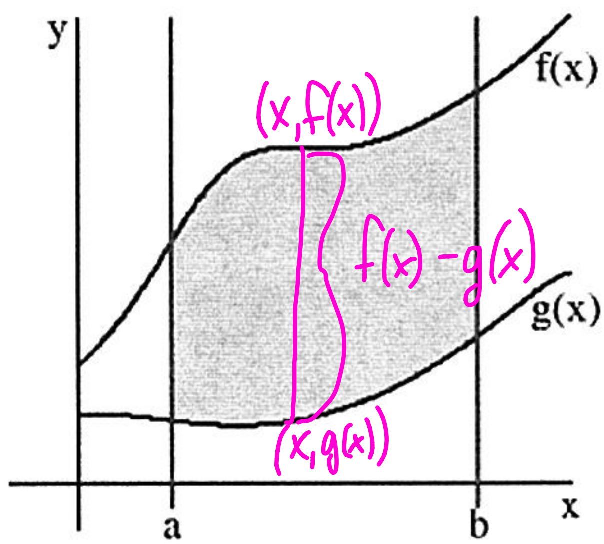
height base

Which can be rewritten as : $Area = \int_a^b f(x)dx$ → base

height

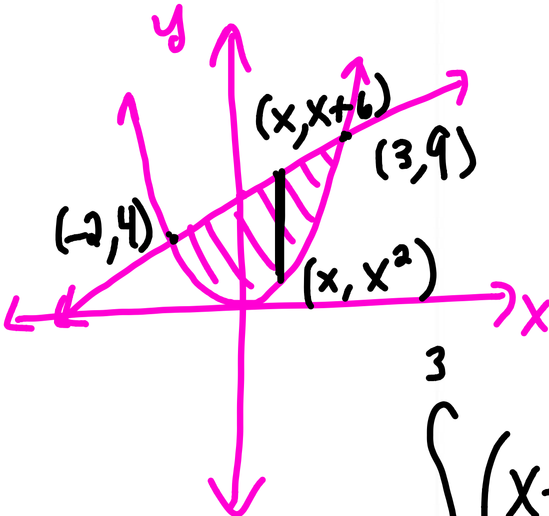
What if the region is not bounded by the x-axis?
What if the area is between 2 curves?

Vertical Strip:



$$Area = \int_{\text{left most x-value}}^{\text{right most x value}} (\text{top curve} - \text{bottom curve})\, dx$$

1. Find the area of the region bounded by $y = x + 6$ and $y = x^2$.

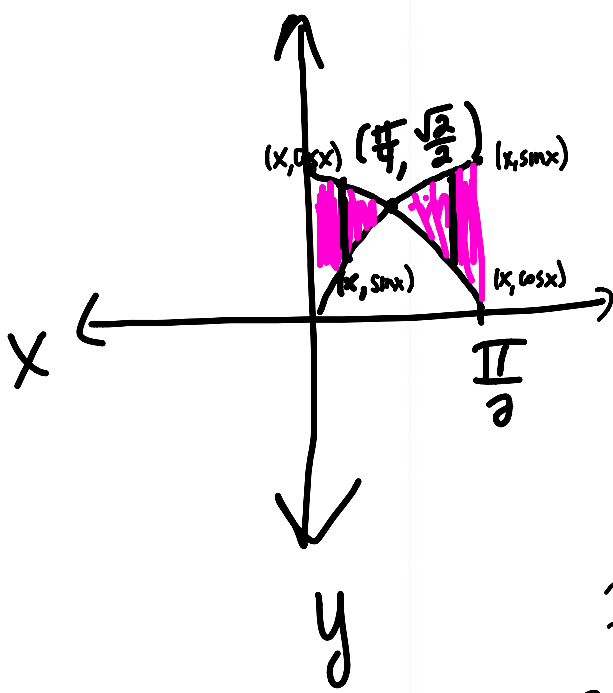


$$x+6=x^2$$
$$0=x^2-x-6$$
$$0=(x-3)(x+2)$$
$$x=3,-2$$

$$\int_{-2}^{3}(x+6-x^2)\,dx = \left.\frac{x^2}{2}+6x-\frac{x^3}{3}\right|_{-2}^{3}$$

$$=\frac{125}{6}$$

2. Find the area of the region bounded by $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$.



$$\sin x = \cos x$$
$$x = \frac{\pi}{4}$$

$$\int_{0}^{\frac{\pi}{4}}(\cos x-\sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)\,dx$$

$$=2\sqrt{2}-2$$

or

$$2\int_{0}^{\frac{\pi}{4}}(\cos x-\sin x)\,dx \text{ or } 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)\,dx$$

Mr. Lee recorded solutions for the 1998 Exam and he kindly shared them with us.

https://www.youtube.com/watch?v=XcCIMLSyUr8 (#s 1-16)
https://www.youtube.com/watch?v=1acRCmjyXAI (#s 17-28)
https://youtu.be/LOSV5tmkDIo. (#s 76-92)