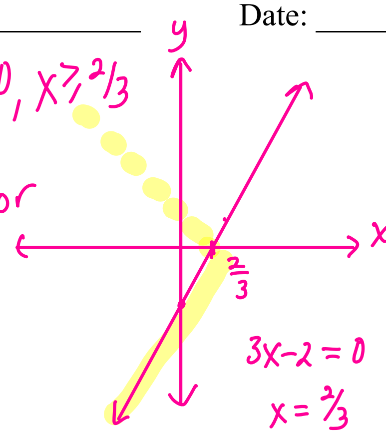
Name: ____________________________________________ Date: ____________

AP Calculus AB

Do now:

$$|3x-2| = \begin{cases} 3x-2 & 3x-2 \geq 0,\ x \geq 2/3 \\ -(3x-2) & ,\ x \leq 2/3 \text{ or} \end{cases}$$




$3x-2=0$

$x = 2/3$

1. $\int_0^3 |3x-2|\,dx =$

$$-\int_0^{2/3} (3x-2)\,dx + \int_{2/3}^{3} (3x-2)\,dx$$

$$\int_{2/3}^{0} (3x-2)\,dx + \int_{2/3}^{3} (3x-2)\,dx = \left.\frac{3x^2}{2} - 2x\right|_{2/3}^{0} + \left.\frac{3x^2}{2} - 2x\right|_{2/3}^{3} = \frac{53}{6}$$

2. Given $h(x) = \begin{cases} x-1 & -1 < x < 2 \\ x & 2 \leq x < 4 \\ x^2 & x \geq 4 \end{cases}$, find $\int_{-1}^{5} h(x)\,dx =$

$$\int_{-1}^{2} (x-1)\,dx + \int_{2}^{4} x\,dx + \int_{4}^{5} x^2\,dx$$

$$\left.\frac{x^2}{2} - x\right|_{-1}^{2} + \left.\frac{x^2}{2}\right|_{2}^{4} + \left.\frac{x^3}{3}\right|_{4}^{5} = \frac{149}{6}$$

Name: ______________________________ Date: ____________

AP Calculus AB: Total Distance Ms. Loughran

Let $s(t)$ represent the position of a particle at time $t$. We know that $s'(t) = v(t)$. In other words,

the derivative of position is velocity.

What does $\int_{t_1}^{t_2} v(t)\,dt$ represent?

$s(t_1) - s(t_2)$

displacement

How would we find the total distance traveled by the particle over the interval $[t_1, t_2]$?

Therefore, total distance = $\int_{t_1}^{t_2} |v(t)dt| \, dt$

If $v \geq 0$,

displacement = total distance

Examples:

1. A particle moves along the $x$-axis according to $s(t) = 2t^3 - 21t^2 + 60t - 14$. Find the total distance traveled from $t = 0$ to $t = 7$.

$v(t) = 6t^2 - 42t + 60$

$$\int_0^7 |6t^2 - 42t + 60| \, dt$$

$6t^2 - 42t + 60 = 0$

$t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$

$$\int_0^2 (6t^2 - 42t + 60)\,dt + \int_5^2 (6t^2 - 42t + 60) + \int_5^7 (6t^2 - 42t + 60)\,dt$$

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+ − + $v(t$

2 5

TD: $\int |v(t)|\,dt$

2. A particle moves along the $x$-axis with acceleration $a(t) = 2t - 3,\ t \geq 0$. At $t = 0$, v=2. Find the total distance traveled from $t = 0$ to $t = 3$.

$$v(t) = \int (2t-3)\,dt = t^2 - 3t + c$$

$v(0) = 2$

$$2 = 0^2 - 3(0) + c$$

$$2 = c$$

$$v(t) = t^2 - 3t + 2$$

$$\int_0^3 |t^2 - 3t + 2|\,dt$$

by hand

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1, 2$

$+ \quad - \quad +$ $v(t)$

$1 \quad 2$

$$\int_0^1 (t^2-3t+2)\,dt + \int_2^1 (t^2-3t+2)\,dt + \int_2^3 (t^2-3t+2)\,dt$$

use calculator:

$$\int_0^3 |t^2 - 3t + 2|\,dt = 1.833 \text{ or } \frac{11}{6}$$

Name:________________________________ Date:_________

AP Calc Ms. Loughran

1982 AB 1

A particle moves along the X-axis in such a way that its acceleration at time $t$ for $t > 0$ is given by $a(t) = \frac{3}{t^2}$. When $t = 1$, the position of the particle is 6 and the velocity is 2.

(a) Write an equation for the velocity, $v(t)$, of the particle for all $t > 0$.

(b) Write an equation for the position, $x(t)$, of the particle for all $t > 0$.

(c) Find the position of the particle when $t = e$.

$a(t) = 3t^{-2}$

$v(t) = \int a(t)\,dt$

$v(t) = -3t^{-1} + C$ (1,2)

$2 = -3(1)^{-1} + C$

$2 = -3 + C$

$5 = C$

(a) $v(t) = -3t^{-1} + 5$

$\frac{-3}{t} + 5$

$x(t) = \int v(t)\,dt$

$\int (-3t^{-1} + 5)\,dt$ (1, 6)

since $t>0$ don't need | |

$x(t) = -3\ln t + 5t + 1$

$x(t) = -3\ln|t| + 5t + C$

$6 = -3\ln|1| + 5(1) + C$

$6 = -3(0) + 5 + C$

$1 = C$

(b) $x(t) = -3\ln|t| + 5t + 1$

(c) $x(e) = -3\ln|e| + 5e + 1$

$-3 + 5e + 1$

$5e - 2$

1998: AB-4

Let $f$ be a function with $f(1) = 4$ such that for all points $(x, y)$ on the graph of $f$ the slope is given by $\dfrac{3x^2 + 1}{2y}$.

(a) Find the slope of the graph of $f$ at the point where $x = 1$.

(b) Write an equation for the line tangent to the graph of $f$ at $x = 1$ and use it to approximate $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find $f(1.2)$.

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(a) $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ and $f(1) = 4 \Longrightarrow \left.\dfrac{dy}{dx}\right|_{(1,4)} = \dfrac{1}{2}$ ☺

(b) The equation of the tangent: $y - 4 = \dfrac{1}{2}(x - 1)$ or $y = \dfrac{1}{2}x + \dfrac{7}{2}$ ☺

$f(1.2) \approx \dfrac{1}{2}(1.2) + \dfrac{7}{2} = 4.1$ ☺

(c) $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y} \Longrightarrow \int 2y\,dy = \int (3x^2 + 1)\,dx \Longrightarrow y^2 = x^3 + x + C$

$f(1) = 4 \Longrightarrow C = 14 \Longrightarrow f(x) = \sqrt{x^3 + x + 14}$ ☺

(d) $f(1.2) \approx 4.114$ ☺

**Rectilinear Motion Revisited Packet**
**For any questions in this packet prior to 2000 do not use the graphing calculator.**
**(Use of graphing calculator on AP exam began in 1995.)**
**Then follow these guidelines:**
**2000-2010 #s 1-3 are calculator active and #s 4-6 are non-calculator**
**2011- present #s 1-2 are calculator active and 3-6 are non-calculator**

**1987 AB1** did on 02-27

A particle moves along the $x$-axis so that its acceleration at any time $t$ is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$, its position is $x(1) = 20$.

(a) Write an expression for the velocity $v(t)$ of the particle at any time $t$.

(b) For what values of $t$ is the particle at rest?

(c) Write an expression for the position $x(t)$ of the particle at any time $t$.

(d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

**1991 BC1**

A particle moves on the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

(a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

(b) Find all values of $t$ for which the particle is at rest.

(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

**1997 AB1**

A particle moves along the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

(b) For what values of $t$, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

1. A particle moves on the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

(a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

(b) Find all values of $t$ for which the particle is at rest.

(c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

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a.) $x(t) = \int v\,dt = 4t^3 - 18t^2 + 15t + C$

$x(1) = 0 = 4 - 18 + 15 + C \Rightarrow C = -1$

$$\boxed{x(t) = 4t^3 - 18t^2 + 15t - 1}$$

b.) Particle is at rest when $v(t) = 0$

$v(t) = 3(4t^2 - 12t + 5) = 3(2t-1)(2t-5)$

$$\boxed{v(t) = 0 \text{ when } t = \tfrac{1}{2}, \tfrac{5}{2}}$$

c.) $v'(t) = 3(8t - 12) \Rightarrow v'(t) = 0$ when $t = \frac{3}{2}$, $0 \leq t \leq 2$

$v'(t)$: on $t$ from $0$ to $\frac{3}{2}$: $-$; from $\frac{3}{2}$ to $2$: $+$

$v(\frac{3}{2})$ is an absol. min. $\Rightarrow v(0)$ or $v(2)$ is an absol. max.

$v(0) = 15$; $v(2) = 3(16 - 24 + 5) = -9$

$$\boxed{v \text{ has an absol. max. of } 15 \text{ at } t = 0}$$

d.)

| $t$ | $x(t)$ | |
|---|---|---|
| 0 | $-1$ | |
| | | 3.5 |
| $\frac{1}{2}$ | $\frac{5}{2}$ | |
| | | 13.5 |
| 2 | $-11$ | |

$\Rightarrow$ $\boxed{\text{Total dist.} = 17}$

OR Total dist. $= \left| \int_0^{1/2} (12t^2 - 36t + 15)\,dt \right| + \left| \int_{1/2}^{2} (12t^2 - 36t + 15)\,dt \right|$