$v(t) = -1 + e^{3-t}$

$v(t)=0 \quad t=1$

2003 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

d)

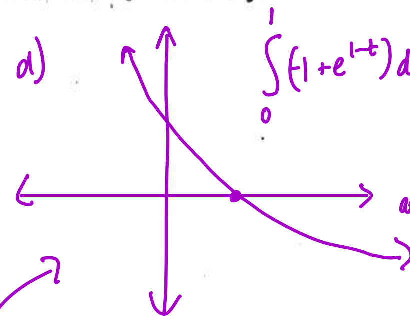
$x(t) = \int (-1 + e^{1-t})\,dt$

$= -t - e^{1-t} + C$

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

d)



$\int_0^1 (-1+e^{1-t})\,dt + \int_1^3 (-1+e^{1-t})\,dt$

and then integrate & evaluate

use graph

OR

4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

   (a) Find the acceleration of the particle at time $t = 3$.

   (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

   (c) Find all values of t at which the particle changes direction. Justify your answer.

   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

d)

$x(0) = -e + \cancel{c}$

$x(1) = -1 - e^0 + c = -2 + \cancel{c}$

$x(3) = -3 - e^{-2} + \cancel{c}$

TD: $|-e-(-2)| + |-2-(-3-e^{-2})|$

(a) $a(t) = -e^{1-t}$ $\quad a(3) = -e^{-2}$ or $-\frac{1}{e^2}$

(b) $v(3) = -1 + e^{-2}$

Since $a(3)$ and $v(3)$ are both negative, the particle is speeding up at $t=3$.

(c) $v(t) = 0$

$-1 + e^{1-t} = 0$

$e^{1-t} = 1$

$1 - t = 0$

$t = 1$

$v(t)$: $+$ on $(0, 1)$, $-$ after 1

$t=\frac{1}{2}$: $-1+e^{\frac{1}{2}}$ $\quad$ $t=2$: $-1+e^{-1}$

$t=1$ b/c $v(t)>0$ before $t=1$ and $v(t)<0$ after $t=1$

(velocity changes sign around $t=1$)

$\frac{70-10}{3} = \frac{60}{3} = 20$

2006 AB4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

$\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80} = \frac{44}{80}\ \text{ft/s}^2$

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$. $= 20(22+35+44)$ ft

change in postion from $t = 10$ to $t = 70$ seconds

since $v(t) > 0$, it actually represents the total distance in ft of the rocket b/w 10 and 70 seconds.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

$v(0) = 2$

$V_B = \int \frac{3}{\sqrt{t+1}}\,dt = 3\int \frac{1}{\sqrt{t+1}}\,dt$

$V_A = 49$ ft/s

$u = t+1$

$du = dt$

$3\int u^{-\frac{1}{2}}\,du$

$v(0) = 2$

$V_B = 3 \cdot 2u^{\frac{1}{2}} + C$

$V_B(t) = 6u^{\frac{1}{2}} + C$

$2 = 6\sqrt{t+1} + C$

$2 = 6\sqrt{0+1} + C$

$2 = 6 + C$

$-4 = C$

$V_B(t) = 6\sqrt{t+1} - 4$

$V_B(80) = 50$ ft/s

Rocket B b/c

$V_B(80) = 50$ ft/s $\quad V_A(80) = 49$ ft/s

Homework 03-05

Name: ______________________________ Date: ______________

AP Calculus AB Homework Ms. Loughran

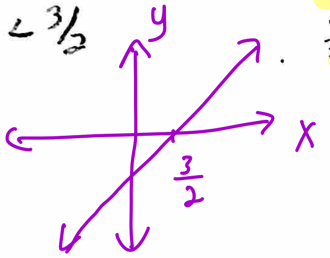
Evaluate each of the following.

1. $\int_{0}^{2} |2x-3|\,dx$

$$-\int_{0}^{3/2} 2x-3\,dx + \int_{3/2}^{2} 2x-3\,dx$$

$$|2x-3| = \begin{cases} 2x-3 & 2x-3 \geq 0,\ x \geq 3/2 \\ -(2x-3) & x < 3/2 \end{cases}$$

or



$$\int_{3/2}^{0} (2x-3)\,dx + \int_{3/2}^{2} (2x-3)\,dx$$

$$x^2-3x\Big|_{3/2}^{0} + x^2-3x\Big|_{3/2}^{2}$$

$$0-\left(\frac{9}{4}-\frac{9}{2}\right) + \left((2^2-6)-\left(\frac{9}{4}-\frac{9}{2}\right)\right)$$

$$-\frac{9}{4} + \frac{9}{2} + 4 - 6 - \frac{9}{4} + \frac{9}{2}$$

$$\frac{18}{2} + \frac{-18}{4} - 2 = \frac{23}{4}$$

$$9 - \frac{18}{4} - 2 = 7 - \frac{18}{4} = \frac{10}{4} = \frac{5}{2}$$

2. $\int_{0}^{3\pi/4} |\cos x|\,dx$

$$\int_{0}^{\pi/2} \cos x\,dx + \int_{3\pi/4}^{\pi/2} \cos x\,dx$$

$$\sin x\Big|_{0}^{\pi/2} + \sin x\Big|_{3\pi/4}^{\pi/2}$$

$$= \sin \pi/2 - \sin 0 + (\sin \pi/2 - \sin 3\pi/4)$$

$$1 - 0 + 1 - \sqrt{2}/2 = 2 - \sqrt{2}/2$$

3. $\int_{-2}^{3} f(x)\,dx$, where $f(x) = \begin{cases} -x, & x \geq 0 \\ x^2, & x < 0 \end{cases}$

$$\int_{-2}^{0} x^2\,dx + \int_{0}^{3} -x\,dx = \frac{x^3}{3}\Big|_{-2}^{0} - \frac{x^2}{2}\Big|_{0}^{3}$$

$$0 - \frac{(-2)^3}{3} - \left(\frac{9}{2} - 0\right) = \frac{(2)8}{(2)3} - \frac{(3)9}{(3)2} = \frac{-11}{6}$$

4. $\int_{-1}^{1} |e^x - 1|\,dx$

$$-\int_{-1}^{0} e^x-1\,dx + \int_{0}^{1} e^x-1\,dx = \int_{0}^{-1} e^x-1\,dx + \int_{0}^{1} e^x-1\,dx$$

$$|e^x-1| = \begin{cases} e^x-1 \\ -(e^x-1) \end{cases} \quad e^x-1 \geq 0 \quad \ln e^x \geq \ln 1 \quad x \geq 0 \text{ OR } x < 0$$

$$e^x - x\Big|_{0}^{-1} + e^x - x\Big|_{0}^{1} = e^{-1}+1-(e^0)+(e^1-1)-(e^0)$$

$$e^{-1} + \cancel{1} - \cancel{1} + e - 1 - 1$$

$$\frac{1}{e} + e - 2$$

AB Calc Even More Rectilinear Motion Problems

1989 AB 3

$a(t) = 4\cos(2t)$

$v(0) = 1$

(a) $\int a(t) = \int 4\cos(2t) = 4\int \cos(2t) = 4 \cdot \frac{1}{2}\int \cos u = 2\sin u + c$

$u = 2t$

$du = 2\,dt$

$v(t) = 2\sin(2t) + c$

$v(0) = 1$

$1 = 2\sin(2(0)) + c$

$1 = 0 + c$

$1 = c$

$v(t) = 2\sin(2t) + 1$

$\int \sin u = -\cos u$

(b) $x(t) = \int v(t) = \int 2\sin 2t + 1 = 2\int \sin 2t\,dt + \int 1\,dt$

$u = 2t$

$du = 2dt$

$x(t) = -\cos 2t + t + C$

$x(0) = 0$ $\quad 0 = -\cos 2(0) + 0 + c$

$0 = -1 + c$

$1 = c$

$x(t) = -\cos 2t + t + 1$

(c) $v(t) = 0$

$2\sin(2t) + 1 = 0$

$2\sin(2t) = -1$

$\sin(2t) = -1/2$

$2t = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad t = \frac{7\pi}{12}, \frac{11\pi}{12}$

1990 AB 1

$a(t)=12t^2-4$ $\quad$ $x(1)=3$ $\quad$ initially at rest $v(0)=0$

(a) $v(t)=\int a(t)=\int 12t^2-4 = 4t^3-4t+c$ $\quad$ $4t^3-4t=0$

$v(0)=0$ $\quad$ $4t^3-4t+0$ $\quad$ $4t(t^2-1)=0$

$\boxed{v(t)=4t^3-4t}$ $\quad$ $t=0 \mid t=\pm 1$

reject -1

$\boxed{@\ t=0,\ t=1}$

(b) $x(t)=\int v(t)=\int 4t^3-4t = t^4-2t^2+c$

$x(1)=3$

$3=1^4-2(1)^2+c$

$3=1-2+c$

$3=-1+c$

$4=c$

$\boxed{x(t)=t^4-2t^2+4}$

(c) total distance $=\int_0^2 |v(t)|\,dt = \int_0^2 4t^3-4t = \boxed{10}$

$v(t)$: $-$ on $(0,1)$, $+$ on $(1,2)$

OR

$x(0)=4$
$x(1)=3$ } 1
$x(2)=12$ } 9

$\boxed{10}$

$-\int_0^1 4t^3-4t + \int_1^2 4t^3-4t$

$\int_1^0 4t^3-4t$

$\frac{4t^4}{4}-\frac{4t^2}{2}$

$t^4-2t^2\Big|_1^0$

$0^4-2(0)^2-(1^4-2(1)^2)+\left[2^4-2(2)^2-(1^4-2(1)^2)\right]$

$8-(-1)$

$16-8$

$0+1+9$

$1+9=10$

2002 AB 3

$v(t) = \sin\left(\frac{\pi}{3}t\right)$

(a) $a(t) = \cos\left(\frac{\pi}{3}t\right) \cdot \frac{\pi}{3}$

$a(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right)$

$a(4) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}\cdot 4\right)$

$a(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$

$a(4) = \frac{\pi}{3}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} = -0.523$

(b) at $t=4$ acc $\ominus$

vel $\ominus$

speed $\uparrow$

Both are correct: I is True $a(t)<0$

II is True, $v(t)<0$ and $a(t)<0$

(c) $\int_0^4 |v(t)|\,dt = 2.387$

$\int_0^4 v(t)\,dt = x(4) - x(0)$

$x(4) = x(0) + \int_0^4 v(t)\,dt$

2+ displacement

(d) $x(t) = \int \sin\left(\frac{\pi}{3}t\right)dt$

$u = \frac{\pi}{3}t$

$\frac{du}{\pi/3} = \frac{\frac{\pi}{3}dt}{\pi/3}$

$x(t) = \frac{3}{\pi}\cdot -\cos\frac{\pi}{3}t + C$

$x(t) = -\frac{3}{\pi}\cos\frac{\pi}{3}t + C$

$x(0) = 2$

$2 = -\frac{3}{\pi}\cos\frac{\pi}{3}(0) + C$

$2 = \left(-\frac{3}{\pi}\right)(1) + C$

$2 = -\frac{3}{\pi} + C$

ON CAC

$C = 2 + \frac{3}{\pi}$

$2 + \frac{9}{2\pi}$

$x(t) = -\frac{3}{\pi}\cos\frac{\pi}{3}t + 2 + \frac{3}{\pi}$

$x(4) = -\frac{3}{\pi}\cos\frac{\pi}{3}(4) + 2 + \frac{3}{\pi}$

$= -\frac{3}{\pi}\cos\frac{4\pi}{3} + 2 + \frac{3}{\pi}$

$= -\frac{3}{\pi}\left(-\frac{1}{2}\right) + 2 + \frac{3}{\pi}$

$= \frac{3}{2\pi} + 2\left(\frac{3}{\pi}\right) + 2$

$= 2 + \frac{9}{2\pi} = 3.432$