Name: ___________________________________ Date: _______
AP Calc Ms. Loughran

Do Now:

Evaluate each.

1. $\frac{d}{dx}\left[\int_4^x t^2\,dt\right] =$ $\frac{d}{dx}\left[\frac{t^3}{3}\Big|_4^x\right]$

$\frac{d}{dx}\left[\frac{x^3}{3} - \frac{4^3}{3}\right] = x^2$

$\frac{d}{dx}\left[\frac{1}{3}x^3 - \frac{64}{3}\right] = x^2$

Thoughts on a shortcut to evaluate this?

2. $\frac{d}{dx}\left[\int_0^{\cos x} t\,dt\right] =$

$\frac{d}{dx}\left[\frac{t^2}{2}\Big|_0^{\cos x}\right] = \frac{d}{dx}\left[\frac{\cos^2 x}{2} - 0\right]$

$= \frac{1}{2}\cdot\cancel{2}\cos x(-\sin x)$

$-\cos x \sin x$

Does your conjecture still hold?

3. $\frac{d}{dx}\left[\int_1^{\tan x} e^t\,dt\right] =$

$\frac{d}{dx}\left[e^t\Big|_1^{\tan x}\right] = \frac{d}{dx}\left[e^{\tan x} - e^1\right]$

$e^{\tan x}\sec^2 x$

AP Calculus AB

THEOREM (*The Fundamental Theorem of Calculus, Part 2*). *If f is continuous on an interval I, then f has an antiderivative on I. In particular, if a is any point in I, then the function F defined by*

$$F(x) = \int_a^x f(t)\,dt$$

is an antiderivative of f on I; that is, $F'(x) = f(x)$ for each x in I, or in an alternative notation

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

If a definite integral has a variable upper limit of integration and a continuous integrand, then the derivative of the integral with respect to its upper limit is equal to the integrand evaluated at the upper limit.

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x))g'(x)$$

In words, *to differentiate an integral with a constant lower limit and a function as the upper limit, substitute the upper limit into the integrand, and multiply by the derivative of the upper limit.*

Name: ________________________________

AP Calculus - Using the Fundamental Theorem of Calculus

1. If $F'(x) = \left(\int_1^{x^4} \sqrt{1+u^3}\,du\right)'$, find $F'(\sqrt[3]{2})$.

$F'(x) = \sqrt{1+(x^4)^3} \cdot 4x^3$

$F'(x) = 4x^3\sqrt{1+x^{12}}$

$F'(\sqrt[3]{2}) = 4(\sqrt[3]{2})^3\sqrt{1+(\sqrt[3]{2})^{12}} = 8\sqrt{17}$

$(2^{\frac{1}{3}})^{12}$

2. Find $\int_0^1 g''(t)\,dt$ where $g'(t) = \left(\int_1^t \sqrt{x^2+1}\,dx\right)'$

$g'(1) - g'(0)$

$g'(t) = \sqrt{t^2+1}$

$\sqrt{1^2+1} - \sqrt{0^2+1}$

$\sqrt{2} - 1$

3. If $f(x) = e^{\int_2^x \frac{t\,dt}{1+t^4}}$ find $f'(2)$.

$f'(x) = e^{\int_2^x \frac{t\,dt}{1+t^4}} \cdot \left(\int_2^x \frac{t\,dt}{1+t^4}\right)'$

$f'(x) = e^{\int_2^x \frac{t\,dt}{1+t^4}} \cdot \frac{x}{1+x^4}$

$f'(2) = e^{\int_2^2 \frac{t\,dt}{1+t^4}} \cdot \frac{2}{1+2^4} = e^0 \cdot \frac{2}{17} = \frac{2}{17}$

4. If $f(t) = \int_{t^2}^4 \frac{dx}{x^8+1}$ find $\int_{-1}^0 f''(t)\,dt$.

$f(t) = -\int_4^{t^2} \frac{dx}{x^8+1}$

$f'(0) - f'(-1)$

$f'(t) = -\frac{1}{(t^2)^8+1} \cdot 2t = -\frac{2t}{t^{16}+1}$

$0 - \left(\frac{2}{2}\right) = -1$

5. If f' is continuous and f passes through (1, 3) and (3, 1), find $\int_1^3 f'(x)\,dx$.

$f(1) = 3$ $\quad f(3) = 1$

$f(3) - f(1)$

$1 - 3 = -2$

6. Evaluate $\int_0^1 \frac{d}{dx}\left(\sqrt{1+x^3}\right)dx$

$(1+x^3)^{\frac{1}{2}}$

$\boxed{\text{Shortcut: } \sqrt{1+1^3} - \sqrt{1+0^3} \quad \sqrt{2} - 1}$

$\int_0^1 \frac{1}{2}(1+x^3)^{-\frac{1}{2}} \cdot 3x^2\,dx$

$\int_0^1 \frac{3x^2}{2\sqrt{1+x^3}}\,dx \quad u = 1+x^3,\ du = 3x^2\,dx \quad \frac{1}{2}\int_1^2 u^{-\frac{1}{2}}\,du = \frac{1}{2} \cdot 2u^{\frac{1}{2}}\Big|_1^2 = \sqrt{2} - 1$

1976 AB 6

(a) Given $\left(5x^3 + 40\right)' = \left(\int_c^x f(t)\,dt.\right)'$

(i) Find $f(x)$. $f(x) = 15x^2$

(ii) Find the value of c.

(b) If $F(x) = \int_x^3 \sqrt{1+t^{16}}\,dt$, find $F'(x)$.

ii

$$5x^3+40 = \int_c^x 15x^2\,dx$$

$$5x^3+40 = 5x^3 \Big|_c^x$$

$$\cancel{5x^3}+40 = \cancel{5x^3} - 5c^3$$

$$40 = -5c^3$$

$$-8 = c^3$$

$$-2 = c$$

$$F(x) = -\int_3^x \sqrt{1+t^{16}}\,dt$$

(b) $F'(x) = -\sqrt{1+x^{16}} \cdot 1$

$$= -\sqrt{1+x^{16}}$$

1991 BC 1

(a) $x(t) = \int 12t^2 - 36t + 15 \, dt$

$x(t) = 12 \cdot t^3/3 - 36 \cdot t^2/2 + 15t + c$

$x(t) = 4t^3 - 18t^2 + 15t + c$

And @ $t=1$ particle is at origin so $x(1) = 0$

$0 = 4(1)^3 - 18(1)^2 + 15(1) + c$

$0 = 4 - 18 + 15 + c$

$0 = 1 + c$

$-1 = c$

$x(t) = 4t^3 - 18t^2 + 15t - 1$

(b) $v(t) = 0$

$12t^2 - 36t + 15 = 0$

$3(4t^2 - 12t + 5) = 0$

$3(2t-1)(2t-5) = 0$

$t = 1/2 \mid t = 5/2$

(c) $v'(t) = 24t - 36$

$24t - 36 = 0$

$24t = 36$

$t = 36/24 = 3/2$

$v(0) = 15$

$v(3/2) = -12$

$v(2) = -9$

candidate test because it is on a closed interval

Maximum velocity is 15 at $x=0$

(d) $x(0) = -1$

$x(1/2) = 2.5$

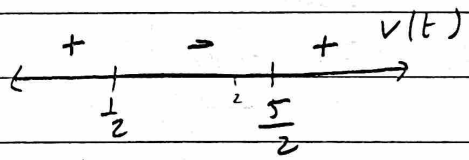
$x(2) = -11$

3.5

13.5

$3.5 + 13.5 = 17$

or



$$\int_0^{1/2} v(t)\,dt + \int_2^{1/2} v(t)\,dt = 17$$

or on calc

$$\int_0^2 |v(t)|\,dt$$

1997 AB 1

(a) $x(t) = \int 3t^2 - 2t - 1 \, dt$

$x(t) = 3 \cdot \frac{t^3}{3} - 2 \cdot \frac{t^2}{2} - t + C$

$x(t) = t^3 - t^2 - t + C$

And $x(2) = 5$

$5 = 2^3 - 2^2 - 2 + C$

$5 = 8 - 4 - 2 + C$

$5 = 2 + C$

$3 = C$

$x(t) = t^3 - t^2 - t + 3$

(b) $\frac{x(3) - x(0)}{3-0} = 3t^2 - 2t - 1$

$\frac{18 - 3}{3-0} = 3t^2 - 2t - 1$

$5 = 3t^2 - 2t - 1$

$-5 \qquad -5$

$0 = 3t^2 - 2t - 6$

$t = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$

$t = \frac{2 \pm \sqrt{76}}{6}$

$t = \frac{2 \pm 2\sqrt{19}}{6} = \frac{1 \pm \sqrt{19}}{3}$

$\frac{1 + \sqrt{19}}{3} \approx 1.786$

(c) $v(t) = 0$

$3t^2 - 2t - 1 = 0$

$(3t+1)(t-1) = 0$

$\cancel{t = -1/3}$ $\quad t = 1$

$x(0) = 3$ } 1

$x(1) = 2$

$x(3) = 18$ } 16

$1 + 16 = 17$

$\int_0^3 |v(t)| \, dt$ area above x-axis - area below x-axis

or $\int_0^1 v(t)\,dt$

$\int_0^1 -v(t)\,dt + \int_1^3 v(t)\,dt$

$-(t^3 - t^2 - t)\Big|_0^1 + t^3 - t^2 - t\Big|_1^3$

$3t^2 - 2t - 1 = 0$

$(3t+1)(t-1) = 0$

$t = -\frac{1}{3} \quad | \quad t = 1$

$+ \quad -\frac{1}{3} \quad - \quad 1 \quad +$

$1 - 1 - 1 \qquad\qquad 15 + 1$

$-(1^3 - 1^2 - 1) - 0 + (27 - 9 - 3 - (1^3 - 1^2 - 1))$

$1 + 16$

17

2004 AB 3

$v(t) = 1 - \tan^{-1}(e^{t}) = Y_1$

$x(0) = -1$

(a) NDeriv(Y_1, X, 2) $= -0.133 = a(2)$

(b) $a(2) < 0$, $v(2) = -.436 < 0$ so speed is increasing at $t=2$.

(c) $v(t) = 0$

Calc the zero

$X = .443$

+ − v(t)

:443

max.

Use trig window maybe to see nicer graph

$v(t) > 0$ for $0 < t < .443$

$v(t) < 0$ for $t > .443$

(d) $X(2) = \int_0^2 v(t) + x(0)$ net area displacement

$-.3606 + -1$

$x(2) = -1.361$ since $v(2) < 0$ and $x(2) < 0$, $x(1) < 0$ particle is moving away from the origin.

2005 AB 5 – no calculator

(a) $\int_0^{24} v(t)\,dt = \frac{(24+12)(20)}{2} = 360$

The car travels 360 meters in these 24 seconds

(b) $v'(4) = \lim_{t\to 4^-} \frac{\overset{20-0}{v(t)-v(4)}}{\underset{4}{t-4}} = 5 \neq 0 = \lim_{t\to 4^+} \frac{\overset{20-20}{v(t)-v(4)}}{\underset{4}{t-4}}$

$v'(20) = \frac{v(24)-v(16)}{24-16} = \frac{0-20}{8} = \frac{-20}{8} = \frac{-5}{2}$ m/sec²

20 is on the line segment, it is not a corner or an edge

(c)
$$v(t) = \begin{cases} 5t, & 0<t<4 \\ 20, & 4<t<16 \\ \frac{-5}{2}t+60, & 16<t<24 \end{cases}$$

Don't really need velocity, just need slopes of velocity

$$a(t) = \begin{cases} 5 & 0<t<4 \\ 0 & 4<t<16 \\ -5/2 & 16<t<24 \end{cases}$$

no = signs b/c there is no acceleration at 4, 16

v is not differentiable there

(d) $\frac{v(20)-v(8)}{20-8} = \frac{10-20}{12} = \frac{-10}{12} = -\frac{5}{6}$ m/sec²

No MVT doesn't apply to v on [8,20]
b/c v is not differentiable at t=16.

(16,20), (24,0)
$m = -5/2$

$y - 0 = -5/2(t-24)$

$y = -5/2\,t + 60$

$v(20) = -5/2(20) + 60 = -50 + 60 = 10$