Name: ___________________________________ Date: _______

AP Calc Ms. Loughran

Do Now:

Evaluate each.

1. $\frac{d}{dx}\left[\int_{4}^{x} t^2 dt\right] =$ $\frac{d}{dx}\left[\frac{t^3}{3}\Big|_{4}^{x}\right] = \frac{d}{dx}\left[\frac{x^3}{3} - \frac{4^3}{3}\right]$

x^2

Thoughts on a shortcut to evaluate this?

2. $\frac{d}{dx}\left[\int_{0}^{\cos x} t\,dt\right] =$ $\frac{d}{dx}\left[\frac{t^2}{2}\Big|_{0}^{\cos x}\right] = \frac{d}{dx}\left[\frac{\cos^2 x}{2} - 0\right]$

$\frac{1}{\cancel{2}} \cdot \cancel{2}\cos x \cdot -\sin x$

$-\cos x \sin x$

Does your conjecture still hold?

3. $\frac{d}{dx}\left[\int_{1}^{\tan x} e^t dt\right] =$ $e^{\tan x} \cdot \sec^2 x$

AP Calculus AB

THEOREM (*The Fundamental Theorem of Calculus, Part 2*). *If f is continuous on an interval I, then f has an antiderivative on I. In particular, if a is any point in I, then the function F defined by*

$$F(x) = \int_a^x f(t)\,dt$$

is an antiderivative of f on I; that is, $F'(x) = f(x)$ for each x in I, or in an alternative notation

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

If a definite integral has a variable upper limit of integration and a continuous integrand, then the derivative of the integral with respect to its upper limit is equal to the integrand evaluated at the upper limit.

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x))g'(x)$$

In words, *to differentiate an integral with a constant lower limit and a function as the upper limit, substitute the upper limit into the integrand, and multiply by the derivative of the upper limit.*

Name: ______________________________

AP Calculus - Using the Fundamental Theorem of Calculus

1. If $F'(x)=\left[\int_1^{x^4}\sqrt{1+u^3}\,du\right]'$, find $F'(\sqrt[3]{2})$.

$F'(x)=\sqrt{1+(x^4)^3}\cdot 4x^3 = 4x^3\sqrt{1+x^{12}}$

$F(2^{\frac13}) = 4(2^{\frac13})^3\sqrt{1+(2^{\frac13})^{12}} = 8\sqrt{17}$

2. Find $\int_0^1 g''(t)dt$ where $g'(t)=\left(\int_1^t\sqrt{x^2+1}\,dx\right)'$.

$g'(1)-g'(0)$ * need $g'(t)=\sqrt{t^2+1}$

$\sqrt{2}-1$

3. If $f(x)=e^{\int_2^x \frac{t\,dt}{1+t^4}}$ find $f'(2)$.

$[e^u]' = e^u\cdot u'$

$f'(x)=e^{\int_2^x\frac{t\,dt}{1+t^4}}\cdot\left[\int_2^x\frac{t\,dt}{1+t^4}\right]' = e^{\int_2^x \frac{t\,dt}{1+t^4}}\cdot\frac{x}{1+x^4}$

$f'(2)=e^{\int_2^2\frac{t\,dt}{1+t^4}}\cdot\frac{2}{1+2^4} = e^0\cdot\frac{2}{17}=\frac{2}{17}$

4. If $f'(t)=\left(\int_{t^2}^4\frac{dx}{x^8+1}\right)'$ find $\int_{-1}^0 f''(t)dt$.

$f'(t)=\left(-\int_4^{t^2}\frac{dx}{x^8+1}\right)'$

$f'(t)= -\frac{1}{t^{16}+1}\cdot 2t = \frac{-2t}{t^{16}+1}$

$f'(0)-f'(-1)$

$0-\frac{2}{2} = -1$

5. If f' is continuous and f passes through (1, 3) and (3, 1), find $\int_1^3 f'(x)dx$. $= f(3)-f(1)$

$f(1)=3$ $f(3)=1$

$1-3$

-2

6. Evaluate $\int_0^1 \frac{d}{dx}\left(\sqrt{1+x^3}\right)dx$ $= \int_0^1 \frac{d}{dx}(1+x^3)^{\frac12}\,dx$

Shortcut

$\sqrt{1+1^3}-\sqrt{1+0^3}$

$\sqrt{2}-1$

$\int_0^1 \frac12(1+x^3)^{-\frac12}\cdot 3x^2\,dx = \frac12\int_0^1 3x^2(1+x^3)^{-\frac12}\,dx$

$u=1+x^3$

$du=3x^2dx$

$\frac12\int_1^2 u^{-\frac12}\,du$

$\frac12\cdot 2u^{\frac12}\Big|_1^2$

$\sqrt{2}-1$

1976 AB 6

(a) Given $5x^3 + 40 = \int_c^x f(t)\,dt$.

(i) Find $f(x)$.

(ii) Find the value of c.

(b) If $F(x) = \int_x^3 \sqrt{1+t^{16}}\,dt$, find $F'(x)$.

Homework 03-05

2003 AB 2

acceleration

$y_2 = \text{nDeriv}(Y_1, X, X)$

$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right) = y_1$

(a) $a(2) = 1.588$

Since $a(2) > 0$ and $v(2) < 0$ the speed is decreasing at $t = 2$.

(b) $v(t) = 0$, $0 < t < 3$

$0 = -(t+1)\sin\left(\frac{t^2}{2}\right)$

$(t+1)$ can never $= 0$

$t \neq -1$

$\sin\left(t^2/2\right) = 0$

$t^2/2 = \pi$

$t^2 = 2\pi$

$t = \sqrt{2\pi}$

OR

on calc $2.5066 \approx 2.506$

(c) $\int_0^3 |v(t)| = 4.333818$ $\quad 4.334$

(d) $x(0) = 1$ displacement FTC

$\int_0^3 v(t)\,dt = s(3) - s(0)$ given

$-2.1971... = s(3) - 1$

$-1.1971... = s(3)$

$A = 2.5066...$

$\int_0^A v(t)\,dt = s(A) - s(0)$

$-3.2654... = s(A) - 1$

$-2.2654... = s(A)$

should store this value

2.506

t	$s(t)$
0	1 given
$\sqrt{2\pi}$	-2.265
3	-1.197

$\therefore$ The greatest distance from the origin is 2.265

1991 BC 1

(a) $x(t) = \int 12t^2 - 36t + 15\,dt$

$x(t) = 12 \cdot t^3/3 - 36 \cdot t^2/2 + 15t + C$

$x(t) = 4t^3 - 18t^2 + 15t + C$

And @ $t=1$ particle is at origin so $x(1) = 0$

$0 = 4(1)^3 - 18(1)^2 + 15(1) + C$

$0 = 4 - 18 + 15 + C$

$0 = 1 + C$

$-1 = C$

$x(t) = 4t^3 - 18t^2 + 15t - 1$

(b) $v(t) = 0$

$12t^2 - 36t + 15 = 0$

$3(4t^2 - 12t + 5) = 0$

$3(2t-1)(2t-5) = 0$

$t = 1/2 \quad | \quad t = 5/2$

(c) $v'(t) = 24t - 36$

$24t - 36 = 0$

$24t = 36$

$t = 36/24 = 3/2$

$v(0) = 15$

$v(3/2) = -12$

$v(2) = -9$

* candidate test because it is on a closed interval *

Maximum velocity is 15 at $x=0$

(d) $x(0) = -1$

$x(1/2) = 2.5$

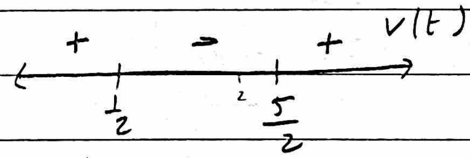
$x(2) = -11$

3.5

13.5

$3.5 + 13.5 = 17$

or



$$\int_0^{1/2} v(t)\,dt + \int_2^{1/2} v(t)\,dt = 17$$

or on calc

$$\int_0^2 |v(t)|\,dt$$

1997 AB 1

(a) $x(t) = \int 3t^2 - 2t - 1 \, dt$

$x(t) = 3 \cdot t^3/3 - 2 \cdot t^2/2 - t + C$

$x(t) = t^3 - t^2 - t + C$

And $x(2) = 5$

$5 = 2^3 - 2^2 - 2 + C$

$5 = 8 - 4 - 2 + C$

$5 = 2 + C$

$3 = C$

$x(t) = t^3 - t^2 - t + 3$

(b) $\frac{x(3) - x(0)}{3 - 0} = 3t^2 - 2t - 1$

$\frac{18 - 3}{3 - 0} = 3t^2 - 2t - 1$

$5 = 3t^2 - 2t - 1$
$-5 \qquad\qquad -5$

$0 = 3t^2 - 2t - 6$

$t = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$

$t = \frac{2 \pm \sqrt{76}}{6}$

$t = \frac{2 \pm 2\sqrt{19}}{6} = \frac{1 \pm \sqrt{19}}{3}$

$(1 + \sqrt{19})/3 \approx 1.786$

(c) $v(t) = 0$

$3t^2 - 2t - 1 = 0$

$(3t + 1)(t - 1) = 0$

$\cancel{t = -1/3}$ $t = 1$

$x(0) = 3$ } 1
$x(1) = 2$
$x(3) = 18$ } 16

$1 + 16 = 17$

$3t^2 - 2t - 1 = 0$

$(3t + 1)(t - 1) = 0$

$t = -\frac{1}{3}$ | $t = 1$

+ $-\frac{1}{3}$ − 1 +

$\int_0^3 |v(t)| \, dt$ area above x-axis − area below x-axis

$\int_0^1 -v(t) \, dt$ (or $\int_0^1 v(t)\,dt$) $+ \int_1^3 v(t) \, dt$

$-(t^3 - t^2 - t)\Big|_0^1 + t^3 - t^2 - t\Big|_1^3$

$-(1^3 - 1^2 - 1) - 0 + (27 - 9 - 3 - (1^3 - 1^2 - 1))$

1-1-1   15 +1

$1 + 16$

17

2004 AB 3

$v(t) = 1 - \tan^{-1}(e^{t}) = Y_1$

$x(0) = -1$

(a) NDeriv(Y_1, X, 2) = -0.133 = $a(2)$

(b) $a(2) < 0$, $v(2) = -.436 < 0$ so speed is increasing at $t=2$.

(c) $v(t) = 0$

Calc the zero

$X = .443$

Use trig window maybe to see nicer graph

\+ − $v(t)$

.443

max.

$v(t) > 0$ for $0 < t < .443$

$v(t) < 0$ for $t > .443$

(d) $X(2) = \int_0^2 v(t) + x(0)$ net area displacement

$-.3606 + -1$

$x(2) = -1.361$ since $V(2) < 0$ and $x(2) < 0$, $x(1) < 0$

∴ particle is moving away from the origin.

2005 AB 5 — no calculator

(a) $\int_0^{24} v(t)\,dt = \frac{(24+12)(20)}{2} = 360$

The car travels 360 meters in these 24 seconds

(b) $v'(4) = \lim_{t\to 4^-} \frac{v(t)-v(4)}{t-4} = 5 \neq 0 = \lim_{t\to 4^+} \frac{v(t)-v(4)}{t-4}$

20 is on the line segment, it is not a corner or an edge

$v'(20) = \frac{v(24)-v(16)}{24-16} = \frac{0-20}{8} = \frac{-20}{8} = \frac{-5}{2}$ m/sec²

(c) $v(t) = \begin{cases} 5t, & 0<t<4 \\ 20 & 4<t<16 \\ \frac{-5}{2}t+60, & 16<t<24 \end{cases}$

Don't really need velocity just need slopes of velocity

$a(t) = \begin{cases} 5 & 0<t<4 \\ 0 & 4<t<16 \\ -5/2 & 16<t<24 \end{cases}$

no = signs b/c there is no acceleration at 4, 16

v is not differentiable there

(d) $\frac{v(20)-v(8)}{20-8} = \frac{10-20}{12} = \frac{-10}{12} = \frac{-5}{6}$ m/sec²

No MVT doesn't apply to v on [8,20] b/c v is not differentiable at t = 16.

(16,20), (24,0)

$m = -5/2$

$y - 0 = -5/2(t-24)$

$y = -5/2\,t + 60$

$v(20) = -5/2(20) + 60 = -50 + 60 = 10$