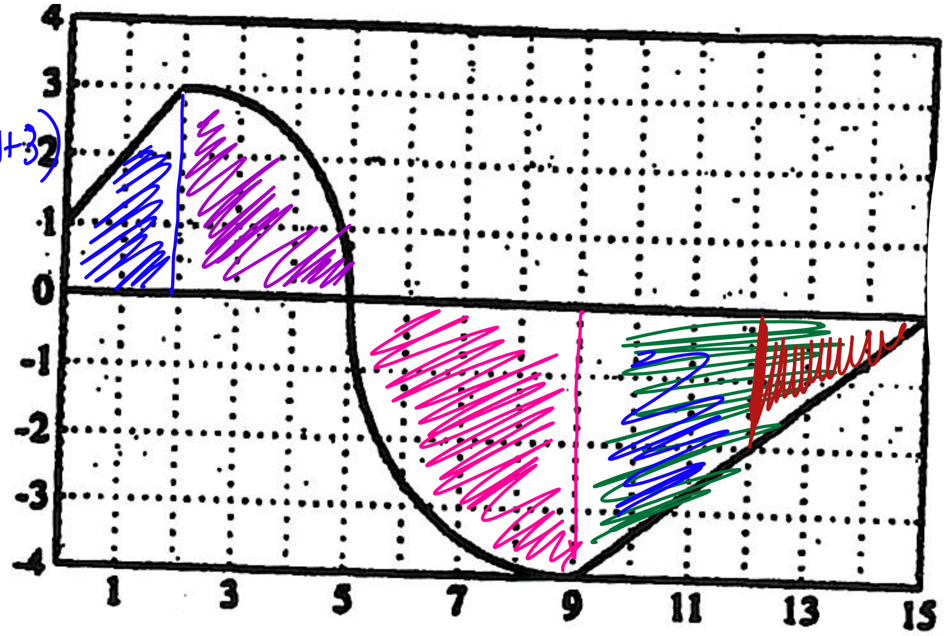
The graph of the function f consists of two line segments and two quarter circles.

Graph of $y = f(x)$



For # 1 – 10, use the graph to evaluate each of the following integrals:

1. $\int_0^2 f(x)\,dx = \frac{1}{2}(2)(1+3)$
   $= 4$

2. $\int_2^5 f(x)\,dx$
   $\frac{1}{4}\pi(3)^2 = \frac{9\pi}{4}$

3. $\int_0^5 f(x)\,dx$
   $4 + \frac{9\pi}{4}$

4. $\int_5^9 f(x)\,dx$
   $-\frac{1}{4}\pi(4)^2 = -4\pi$

5. $\int_5^5 f(x)\,dx = 0$

6. $\int_0^{15} f(x)\,dx$
   $4 + \frac{9\pi}{4} - 4\pi - \frac{1}{2}(6)(4)$
   $4 + \frac{9\pi}{4} - 4\pi - 12$

7. $\int_0^{15} |f(x)|\,dx$
   $4 + \frac{9\pi}{4} + 4\pi + 12$

8. $\int_{15}^9 f(x)\,dx = -\int_9^{15} f(x)\,dx = -(-12)$
   12

9. $\int_{12}^{15} f(x)\,dx = -\frac{1}{2}(3)(2) = -3$

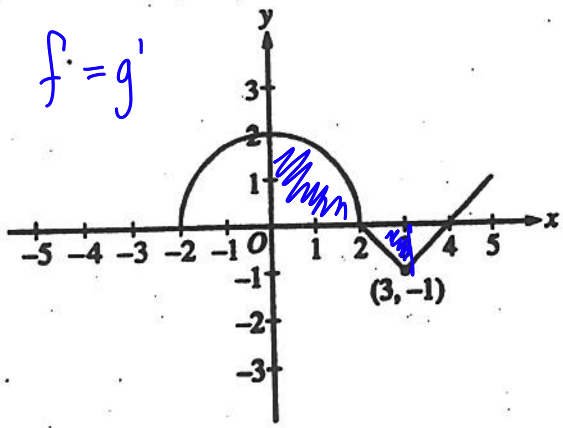
10. $\int_{12}^9 f(x)\,dx = -\int_9^{12} f(x)\,dx =$
    $-\left(-\frac{1}{2}(3)(2+4)\right)$
    9

Name: ______________________

AP Calc AB Using Fundamental Thm of Calc

1997: AB-5; BC-5

$f = g'$



a) $g(3) = \int_0^3 f(x)\,dx$

$= \frac{1}{4}\pi(2)^2 - \frac{1}{2}(1)(1)$

$= \pi - \frac{1}{2}$

The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g'(x) = \left(\int_0^x f(t)\,dt.\right)'$

$g'(x) = f(x)$

(a) Find $g(3)$.

(b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

at $x=2$ b/c $g'(x)$, $f(x)$, changes sign from + to − at $x=2$

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

$g(3) = \pi - \frac{1}{2}$, $(3, \pi - \frac{1}{2})$ $g'(3) = f(3) = -1$ $y - \pi + \frac{1}{2} = -(x-3)$

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

g'' + − or − +

g' ↗ ↘ or ↘ ↗

$x = 0, 3$

Name:________________________________ Date:________

Calculus AB: Applications of the FTC Part 2 Ms. Loughran

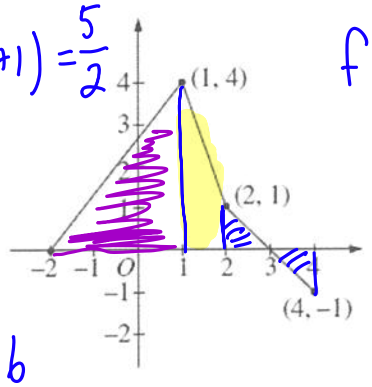
1999 AB 5

a) $g(4)=\int_1^4 f(t)\,dt=\frac{1}{2}(1)(4+1)=\frac{5}{2}$

$f=g'$

$g(-2)=\int_1^{-2} f(t)(dt)=-\int_{-2}^{1} f(t)\,dt$

$-\frac{1}{2}(3)(4)=-6$



. The graph of the function f, consisting of three line segments, is given above. Let $g(x)=\int_1^x f(t)\,dt$.

$\left(\int_1^x f(t)\,dt\right)'$

$g'(x)=f(x)$

(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x=1$.

$g'(1)=f(1)=4$

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x=1$ and $x=2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

(c) There are no relative minimums so the absolute minimum must be at an endpoint.

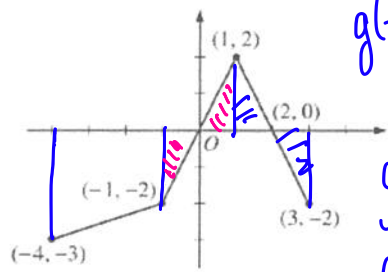
$g(-2)=-6$

$g(4)=\frac{5}{2}$

abs minimum is -6

(d) There is a point of inflection at $x=1$ b/c g' changes from $\nearrow$ to $\downarrow$ around $x=1$. There is no point of inflection at $x=2$ b/c g' does not change from $\nearrow$ to $\searrow$ or $\downarrow$ to $\nearrow$ at $x=2$.

2005 AB 4 Form B



Graph of f

a) $g(-1) = \int_{-4}^{-1} f(t)\,dt = -\frac{1}{2}(3)(2+3) = \frac{-15}{2}$

$g'(-1) = f(-1) = -2$

$g''(-1) = \text{dne}$

The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

$g'(x) = f(x)$

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

g'' + − 0r − +
g' ↗ ↘ ↘ ↗ $x = 1$

At $x=1$ b/c $g'(x)$ changes from ↗ to ↘ around $x=1$

(c) Let h be the function given by $h'(x) = \left(\int_{x}^{3} f(t)\,dt\right)'$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

$x = 3, x = 1, x = -1$

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

$h'(x) = \left(-\int_{3}^{x} f(t)\,dt\right)'$

h' $\ominus$

$f(x)$ $\oplus$

$(0, 2)$

$h'(x) = -f(x)$

$-f(x)$ is $\ominus$ when $f(x)$ +

Homework 03-06

Name: ______________________________ Date: __________

AP Calculus AB: FTC PII Homework Ms. Loughran

For questions 1-3, find the derivative using the FTC Part II and check your answer by evaluating the integral and then differentiating.

1. $\frac{d}{dx}\int_1^{x^3}\frac{1}{t}dt$ $=$ $\frac{1}{x^3}\cdot 3x^2 = \frac{3x^2}{x^3} = \frac{3}{x}$

2. $\frac{d}{dx}\int_1^{\ln x} e^t dt$ $=$ $e^{\ln x}\cdot\frac{1}{x} = 1$

3. $\frac{d}{dx}\int_1^{x^2} t\sqrt{1+t}\,dt$ $x^2\sqrt{1+x^2}\cdot 2x = 2x^3\sqrt{1+x^2}$

4. Let $F(x)=\int_0^x \frac{\cos t}{t^2+3}dt$. Find:
   (a) $F(0) = 0$ (b) $F'(0) = \frac{1}{3}$ (c) $F''(0)$

$\left(\frac{\cos x}{x^2+3}\right)'$ $\frac{(x^2+3)(-\sin x) - \cos x(2x)}{(x^2+3)^2} = \frac{0}{9}$ $0.$

5. Find $\frac{d}{dx}\int_x^1 \sin(t^2)dt$ $-\frac{d}{dx}\int_1^x \sin(t^2)\,dt = -\sin(x^2)$

6. Find $\frac{d}{dx}\int_{\tan x}^3 \frac{t^2}{1+t^2}dt$ $= -\frac{d}{dx}\left[\int_3^{\tan x}\frac{t^2}{1+t^2}dt\right] = -\frac{\tan^2 x}{1+\tan^2 x}\cdot\sec^2 x$ $= -\frac{\tan^2 x\,\cancel{\sec^2 x}}{\cancel{\sec^2 x}} = -\tan^2 x$

7. Find $\frac{d}{dx}\int_1^x \sin(\sqrt{t})dt$ $= \sin\sqrt{x}$

8. Find $\frac{d}{dx}\int_x^0 \frac{t}{\cos t}dt$ $\frac{d}{dx}\left[-\int_0^x \frac{t}{\cos t}dt\right] = -\frac{x}{\cos x}$

9. Let $F(x)=\int_2^x \sqrt{3t^2+1}\,dt$. Find: $F'(x)=\sqrt{3x^2+1}$ $F''(x)=\left((3x^2+1)^{\frac{1}{2}}\right)' = \frac{1}{2}(3x^2+1)^{-\frac{1}{2}}\cdot 6x$ $\frac{3x}{\sqrt{3x^2+1}}$
   (b) $F(2) = 0$ (b) $F'(2) = \sqrt{13}$ (c) $F''(2) = \frac{6}{\sqrt{13}}$

10. Find $\int_0^e \frac{dx}{x+e}$

$u = x+e$
$du = dx$

$\int_e^{2e} u^{-1}du = \ln|u|\Big|_e^{2e} = \ln 2e - \ln e$
$\ln\frac{2e}{e} = \ln 2$