Do Now:

2003 AB 5 Form B

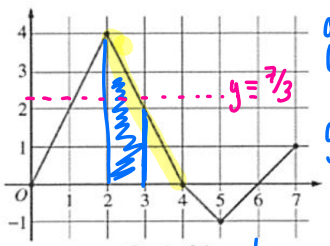
$g'(x) = f(x)$

$g'(3) = f(3) = 2$

$g''(3) = -2$

a) $g(3) = \int_2^3 f(t)\,dt$

$= \frac{1}{2}(1)(2+4) = 3$



Graph of f $= g'$

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

$g'(c) = 7/3$ $\quad f(c) = 7/3$ 2 The graph f would intersect the line $y = 7/3$ twice.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

b) $\frac{1}{3-0}\int_0^3 g'(x)\,dx = \frac{1}{3}\int_0^3 f(t)\,dt = \frac{1}{3}(3+4) = \frac{7}{3}$

d) g'' + to − or − +

g' ↗ ↘ or ↘ ↗ $\quad x = 2, 5$

Homework 03-08

Average Value of a Function HW

1. $f(x) = 3x$; $[1,3]$

$$f_{ave} = \frac{1}{3-1}\int_1^3 3x\,dx = \frac{1}{2}\cdot\frac{3x^2}{2}\Big]_1^3 = \frac{1}{2}\left(\frac{3(3)^2}{2} - \frac{3(1)^2}{2}\right) = \frac{1}{2}\left(\frac{27}{2} - \frac{3}{2}\right) = 6$$

$\frac{27}{2}$ 13

2. $f(x) = \sin x$ $[0,\pi]$

$$f_{ave} = \frac{1}{\pi-0}\int_0^\pi \sin x\,dx = \frac{1}{\pi}\cdot -\cos x\Big]_0^\pi = \frac{1}{\pi}\left(-\cos\pi + \cos 0\right)$$

$-(-\cos 0)$

$$= \frac{1}{\pi}\left(-(-1)+1\right) = 2/\pi$$

3. $f(x) = \frac{1}{x}$, $[1,e]$

$$f_{ave} = \frac{1}{e-1}\int_1^e \frac{1}{x}\,dx = \frac{1}{e-1}\cdot \ln|x|\Big]_1^e = \frac{1}{e-1}\left(\ln e - \ln 1\right)$$

$$\frac{1}{e-1}(1-0) = \frac{1}{e-1}$$

4. $f(x) = x^2$ over $[0,2]$

(a) $$f_{ave} = \frac{1}{2-0}\int_0^2 x^2\,dx = \frac{1}{2}\cdot\frac{x^3}{3}\Big]_0^2 = \frac{1}{2}\left(\frac{2^3}{3} - \frac{0^3}{3}\right) = \frac{1}{2}\left(\frac{8}{3}\right) = \frac{8}{6} = \frac{4}{3}$$

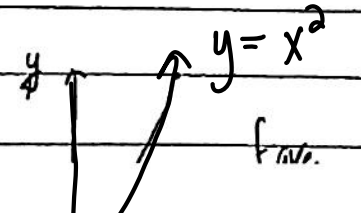
(b) $f(x^*) = f_{ave}$

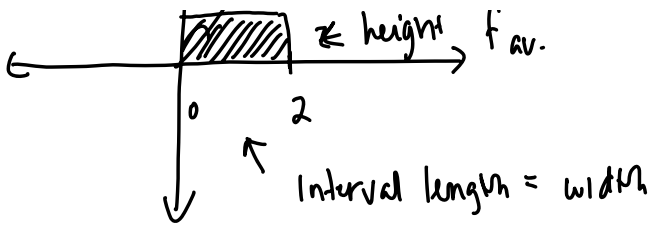
$f(x^*) = \frac{4}{3}$

$x^2 = \frac{4}{3}$

$x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

reject $-\frac{2}{\sqrt{3}}$ blc it's not in $[0,4]$

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