Do Now

① If $f(x) = \int_0^{\cos x} \sqrt{1+t^3}\, dt$, find $f'\left(\frac{\pi}{2}\right)$

$$f'(x) = \sqrt{1+\cos^3 x} \cdot -\sin x$$

$$f'\left(\frac{\pi}{2}\right) = \sqrt{1+\cos^3\left(\frac{\pi}{2}\right)} \cdot -\sin\frac{\pi}{2}$$ FTC Pt 2

$$= \sqrt{1}\,(-1) = -1$$

② Find the average value of $y = x^3\sqrt{x^4+9}$ on the interval $[0, 2]$.

$$\frac{1}{2-0}\int_0^2 x^3\sqrt{x^4+9}\, dx$$

$u = x^4+9$

$du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$$\frac{1}{2}\cdot\frac{1}{4}\int_9^{25} u^{\frac{1}{2}} du = \frac{1}{\cancel{8}_4}\cdot\frac{\cancel{2}}{3}u^{3/2}\Big|_9^{25}$$

$$\frac{1}{12}\left[25^{3/2} - 9^{3/2}\right] = \frac{1}{12}[125-27]$$

3. Let function f be continuous and decreasing, with values as shown in the table:

$.7 \quad .3 \quad .5 \quad .6 \quad .4$

x	2.5	3.2	3.5	4.0	4.6	5.0
$f(x)$	7.6	5.7	4.2	3.8	2.2	1.6

Use the trapezoid method to estimate the area between f and the x-axis on the interval $2.5 \le x \le 5.0$.

$$\frac{1}{2}\left[.7(7.6+5.7)+.3(5.7+4.2)+.5(4.2+3.8)+ .6(3.8+2.2)+.4(2.2+1.6)\right]$$

4. If $\frac{dy}{dx} = -10y$ and if $y = 50$ when $x = 0$, then $y =$

$dy = -10y\,dx$

$\int \frac{dy}{y} = \int -10\,dx$

$\ln|y| = -10x + C \qquad (0,50)$

$\ln 50 = C$

$\ln|y| = -10x + \ln 50$

$e^{\ln|y|} = e^{-10x+\ln 50}$

$|y| = e^{-10x} \cdot e^{\ln 50}$

$|y| = 50e^{-10x}$

$y = \cancel{\pm} 50e^{-10x}$

b/c it has to contain (0,50)

5. If $\frac{dy}{dx} = \cos x \sin^2 x$ and $y = 1$ when $x = \frac{\pi}{2}$, what is y when $x = 0$.

$$\int dy = \int \cos x \sin^2 x \, dx$$

$u = \sin x$
$du = \cos x \, dx$

$$y = \int u^2 \, du$$

$$y = \frac{u^3}{3} + C$$

$$y = \frac{\sin^3 x}{3} + C$$

$(\frac{\pi}{2}, 1)$

$$1 = \frac{\sin^3(\frac{\pi}{2})}{3} + C$$

$$1 = \frac{1}{3} + C$$

$$\frac{2}{3} = C$$

$$y = \frac{\sin^3 x}{3} + \frac{2}{3}$$

when $x = 0$

$$y = \frac{\sin^3 0}{3} + \frac{2}{3}$$

$$y = \frac{2}{3}$$

6

a) $\int_0^3 g'(x)\,dx = g(3) - g(0)$

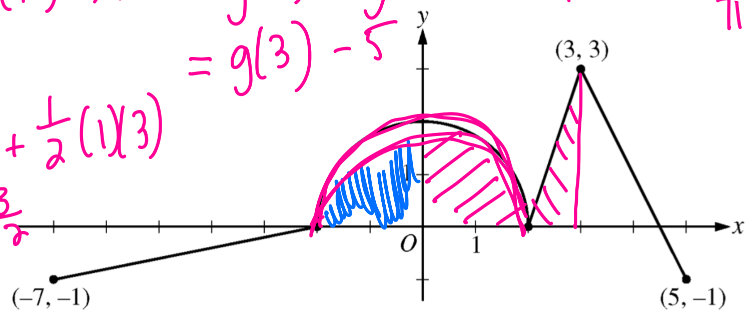
$= g(3) - 5$

$\frac{1}{4}\pi(2)^2 + \frac{1}{2}(1)(3)$

$\pi + \frac{3}{2}$

$\pi + \frac{3}{2} = g(3) - 5$

$\pi + \frac{3}{2} + 5 = g(3)$



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

$x = 0, 2, 3$ b/c g' changes either from ↗ to ↘ to ↘ to ↗ around these x values

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

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a) $\int_{-2}^{0} g'(x)\,dx = g(0) - g(-2)$

$\pi = 5 - g(-2)$

$g(-2) = 5 - \pi$

c) $h(x) = g(x) - \frac{1}{2}x^2$

$h'(x) = g'(x) - x$

$0 = g'(x) - x$

$x = g'(x)$

$x = 3$

$x = \sqrt{2}$

at $\sqrt{2}$ there is a rel max b/c h' changes from + to − and at 3 there is no rel min or max b/c there is no sign change in $h'(x)$

$h(x)$: ↗ ↘ ↘

$h'(x)$: + − −

$\sqrt{2}$ 3

test $x=10$, $g'(0) - 0$, $2 - 0$

$x = 2$, $g'(2) - 2$, $0 - 2$

$x = 4$, $g'(4) - 4$, −

$(0,0)$ $r = 2$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

$\sqrt{4 - x^2} = x$

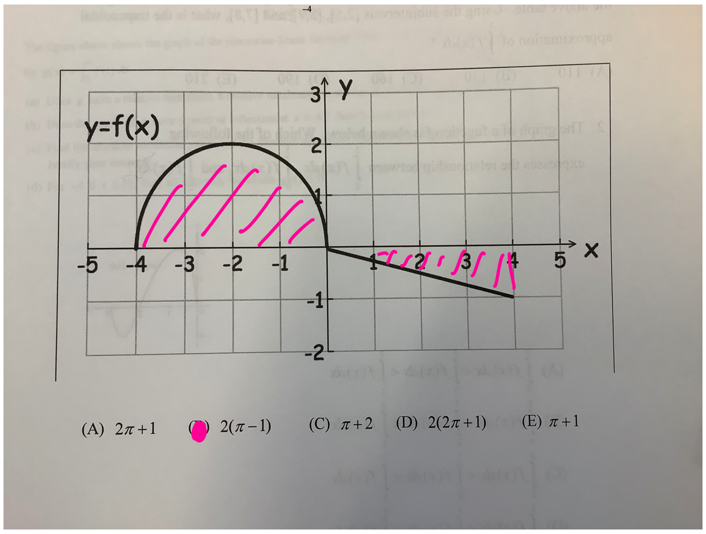
$4 - x^2 = x^2$

$4 = 2x^2$

$2 = x^2$

$\pm\sqrt{2} = x$

⑦



The graph of the function f, shown to the left, consists of a semicircle and a straight line segment. Find $\int_{-4}^{4} f(x)\,dx$

$$\frac{1}{2}\pi(2)^2 - \frac{1}{2}(4)(1)$$

$$2\pi - 2$$

⑧ If $f(x) = \begin{cases} 2x+3 & x<0 \\ 4 & x=0 \\ 1+\sin \pi x & x>0 \end{cases}$, find $\int_{-2}^{1} f(x)\,dx$

$$\int_{-2}^{0} (2x+3)\,dx + \int_{0}^{1} (1+\sin \pi x)\,dx$$

$$x^2 + 3x \Big|_{-2}^{0} + x - \frac{1}{\pi}\cos \pi x \Big|_{0}^{1}$$

$$0 - (4-6) + 1 - \frac{1}{\pi}\cos \pi - \left(0 - \frac{1}{\pi}\cos \pi 0\right)$$

$$2 + 1 + \frac{1}{\pi} + \frac{1}{\pi} = 3 + \frac{2}{\pi}$$

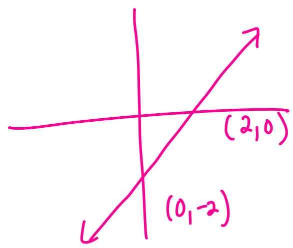
9) If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{-5}^{5} f(x)\,dx = -13$,

find $\int_{5}^{2} f(x)\,dx = -4$

$$\int_{-5}^{2} + \int_{2}^{5} = \int_{-5}^{5} \qquad \int_{2}^{5} = 4$$

$$-17 + \int_{2}^{5} = -13$$

10) $\int_{0}^{3} |x-2|\,dx$



$$\int_{0}^{2} -(x-2)\,dx + \int_{2}^{3} (x-2)\,dx$$

$$\int_{2}^{0} (x-2)\,dx + \int_{2}^{3} (x-2)\,dx$$

$$\frac{x^2}{2} - 2x \Big|_{2}^{0} + \frac{x^2}{2} - 2x \Big|_{2}^{3} = 2.5$$

11) $$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx =$$

$$\int_0^{\frac{\pi}{4}} e^{\tan x}\sec^2 x\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\int_0^1 e^u\,du = e^u\Big|_0^1$$

$$e^1 - e^0 = e - 1$$

12) If $\int_a^b f(x)\,dx = a + 2b$, find $\int_a^b (f(x) + 5)\,dx$

$$\int_a^b f(x)\,dx + \int_a^b 5\,dx$$

$$a + 2b + 5x\Big|_a^b$$

$$a + 2b + 5b - 5a$$

$$7b - 4a$$

(13) $\int \frac{1}{x^2+6x+10}\,dx$

$\underbrace{x^2+6x+9}-9+10$

$(x+3)^2+1$

$\int \frac{1}{(x+3)^2+1}\,dx$

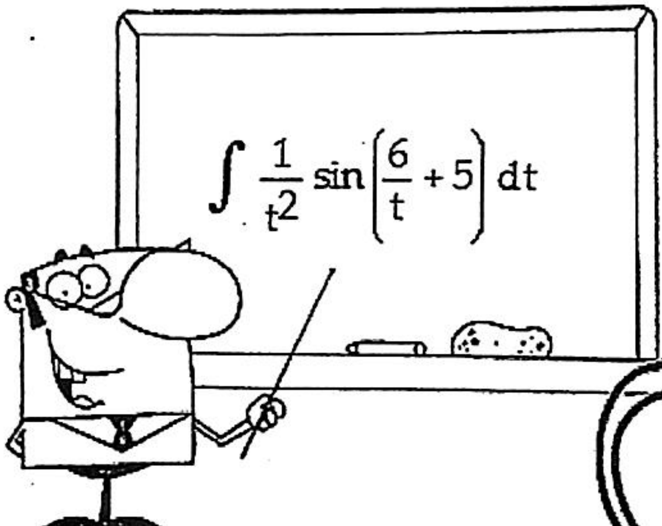
$u = x+3$

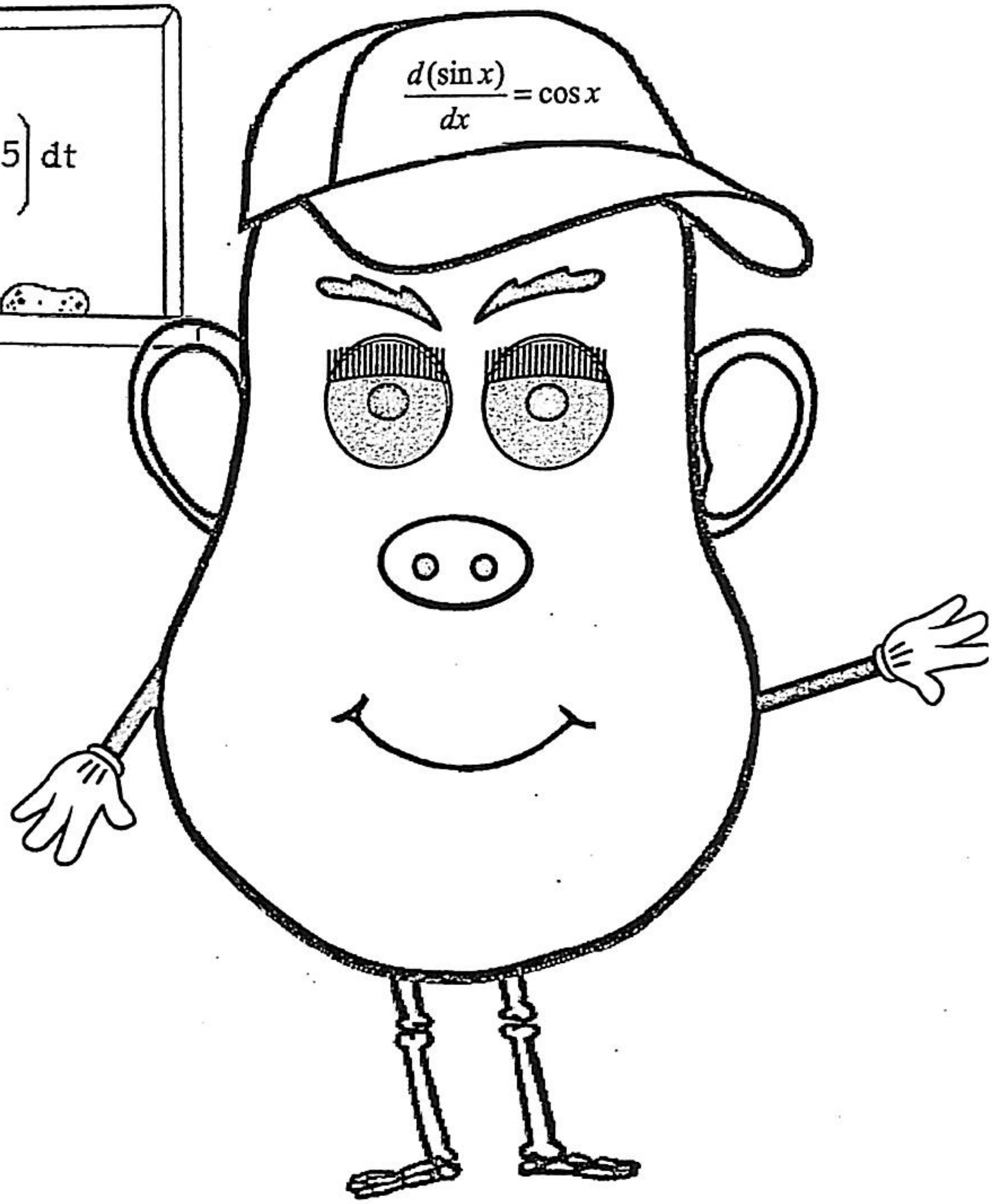
$du = dx$

$\int \frac{1}{u^2+1}\,du = \arctan u + C$

$\arctan(x+3)+C$

Classwork/Homework 03-11





Answer Key
VERSION 2

1. B
2. C
3. B
4. C
5. C
6. D
7. A
8. C
9. B

① $\int x\cos(2x^2)\,dx$

$u = 2x^2$
$du = 4x\,dx$

$\frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin(2x^2) + C$ (B)

② $\int \left(4-\sin\left(\frac{t}{2}\right)\right)^2 \cos\left(\frac{t}{2}\right) dt$

$u = 4-\sin\left(\frac{t}{2}\right)$
$du = -\frac{1}{2}\cos\frac{t}{2}\,dt$

$-2\int u^2\,du$ $\quad -2\frac{u^3}{3} + C$

$-2\frac{\left(4-\sin\left(\frac{t}{2}\right)\right)^3}{3} + C$

(C)

③ $\int 12(4x-3)^{-5}\,dx$

$u = 4x-3$
$du = 4dx$
$\frac{du}{4} = dx$

$3\int u^{-5}\,du = \frac{3u^{-4}}{-4} + C$

$= \frac{-3}{4}(4x-3)^{-4} + C$ (B)

④ $\int x^4(x^5-9)^3\,dx$

$u = x^5-9$
$du = 5x^4\,du$

$\frac{1}{5}\int u^3\,du = 5\frac{u^4}{4} + C$ (C)

$\frac{1}{5}\cdot\frac{(x^5-9)^4}{4} + C$

⑤ $\int \frac{8s^3\,ds}{\sqrt{5-s^4}}$

$-2\int u^{-\frac{1}{2}}\,du$

$u = 5-s^4$
$du = -4s^3\,du$
$\frac{du}{-4} = s^3\,du$

$-2\cdot 2u^{\frac{1}{2}} + C$
$-4\sqrt{5-s^4} + C$

(C)

⑥ $\int 21(y^6+2y^3+4)^3(2y^5+2y^2)\,dy$

$u = y^6+2y^3+4$
$du = 6y^5+6y^2\,dy$
$du = 3(2y^5+2y^2)\,dy$
$\frac{du}{3} = (2y^5+2y^2)\,dy$

(D)

$7\int u^3\,du = \frac{7u^4}{4} + C$

$$\int \frac{dx}{\sqrt{7x+6}}$$

(7) $u = 7x+6$

$\frac{du}{7} = \frac{7dx}{7}$

$\frac{1}{7}\int u^{-\frac{1}{2}} du$

$\frac{1}{7} \cdot 2u^{\frac{1}{2}} + C$

(A) .

(8) $\int \csc^2 6\theta \cot 6\theta \, d\theta$

$u = \cot 6\theta$

$du = -6\csc^2 6\theta$

$\frac{du}{-6} = \csc^2 6\theta$

$-\frac{1}{6}\int u \, du$

$-\frac{1}{6} \frac{u^2}{2} + C$

$-\frac{1}{12}(\cot 6\theta)^2 + C$ (C).

(9) $\int \frac{1}{t^2} \sin\left(\frac{6}{t}+5\right) dt$

$u = 6t^{-1} + 5$

$du = \frac{-6}{t^2} dt$

$\frac{du}{-6} = \frac{1}{t^2} dt$

$-\frac{1}{6}\int \sin u \, du$

$+\frac{1}{6}\cos u + C$ (B) .

$\frac{1}{6}\cos\left(\frac{6}{t}+5\right) + C$