Do Now:

2012 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)(\frac{1}{2}) = -\frac{1}{4}$

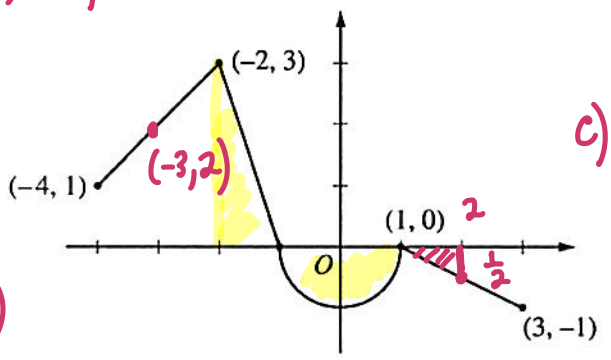
$g(-2) = -\int_{-2}^{1} f(t)\,dt =$

$-\left(\frac{1}{2}(1)(3) - \frac{\pi(1)^2}{2}\right)$

$-\left(\frac{3}{2} - \frac{\pi}{2}\right)$

b) $g'(-3) = f(-3) = 2$

$g''(-3) = f'(-3) = 1$



Graph of f

c)

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$. $g'(x) = f(x)$

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers. $g'(x) = 0$

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

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c) at $x = -1$, rel max b/c $g'(x)$ changes sign from + to − around $x = -1$

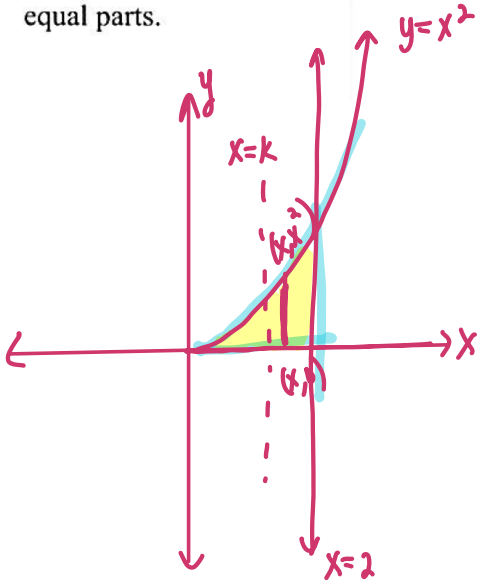
at $x = 1$ neither b/c $g'(x)$ doesn't change sign around $x = 1$

d) g'' − to + or + to −

g' ↓ to ↗ or ↗ to ↓ $x = -2, 1, 0$

Finishing up from yesterday…

6. Find a vertical line $x = k$ that divides the area enclosed by $x = \sqrt{y}$, $x = 2$ and $y = 0$ into 2 equal parts.



$x^2 = y$ half of parabola graph

$x = \pm\sqrt{y}$

$$\int_0^2 (x^2 - 0)\,dx = 2\int_0^k (x^2 - 0)\,dx$$

$$\frac{x^3}{3}\Big|_0^2 = 2\left[\frac{x^3}{3}\Big|_0^k\right]$$

$$\frac{8}{3} = \frac{2k^3}{3}$$

$$8 = 2k^3$$

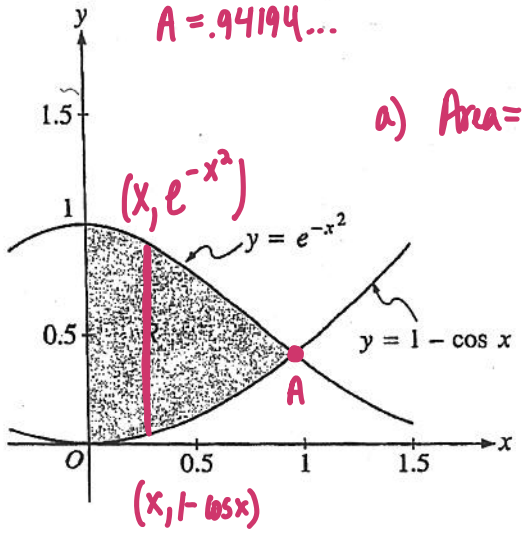
$$4 = k^3$$

$$\sqrt[3]{4} = k$$

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



A = .94194...

a) Area $= \int_0^A \left(e^{-x^2} - (1-\cos x)\right) dx$

$= .591$

or

.590

(x, e^{-x^2})

$(x, 1-\cos x)$

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

GO ON TO THE NEXT PAGE.

2001 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

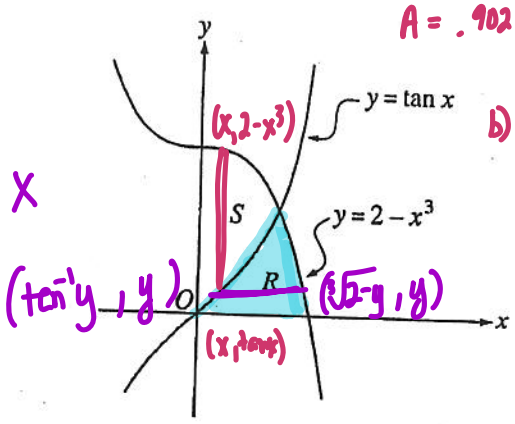
$A = .9021\ldots$

$y = 2 - x^3$

$\sqrt[3]{2-y} = x$

a)

to be continued



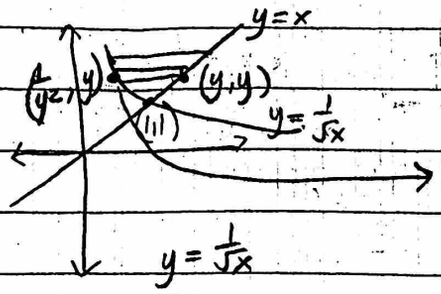
b) $\text{Area}_S = \int_0^A \left((2-x^3) - \tan x\right) dx$

$= 1.161$ or 1.160

1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.
   (a) Find the area of R.
   (b) Find the area of S.
   (c) Find the volume of the solid generated when S is revolved about the x-axis.

Homework 03-13

3) $y=x$, $y=\frac{1}{\sqrt{x}}$



$x=\frac{1}{\sqrt{x}}$

$x\sqrt{x}=1$

$x^3=1$

$x=1$

$(1,1)$

$y=\frac{1}{\sqrt{x}}$

$\sqrt{x}\,y=1$

$\sqrt{x}=\frac{1}{y}$

$x=\frac{1}{y^2}$

$\frac{1}{\sqrt{x}}=2$

$2\sqrt{x}=1$

$\sqrt{x}=\frac{1}{2}$

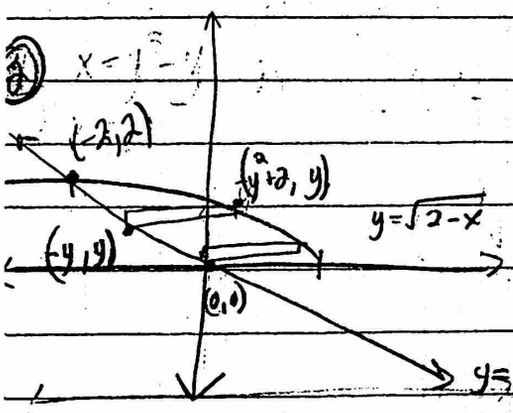
$x=\frac{1}{4}$

$$\int_1^2 \left(y-\frac{1}{y^2}\right)dy = \frac{1}{2}y^2\Big]_1^2 - \left(-\frac{1}{y}\right]_1^2$$

$$\frac{1}{2}(2)^2-\frac{1}{2}(1)^2-\left(-\frac{1}{2}-(-1)\right)$$

$$2-\frac{1}{2}+\frac{1}{2}-1=1$$

3) $x=1$



$-x=\sqrt{2-x}$

$x^2=2-x$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \quad x=1$

$y=2 \quad y=-1$

$y=\sqrt{2-x}$

$y^2=2-x$

$y^2-2=-x$

$-y^2+2=x$

$y=-x$

$-y=x$

$$\int_0^2 -y^2+2-(-y)\,dy$$

$$\int_0^2 -y^2+2+y\,dy$$

$$\int_0^2 -y^2dy+\int_0^2 2\,dy+\int_0^2 y\,dy$$

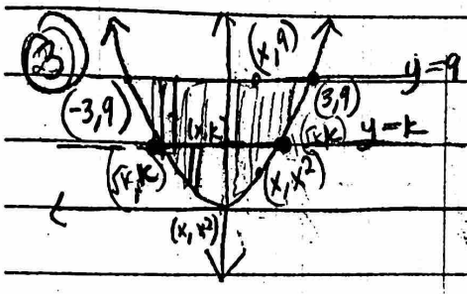
$$-\frac{1}{3}y^3\Big]_0^2+2y\Big]_0^2+\frac{1}{2}y^2\Big]_0^2$$

$$-\frac{1}{3}(2)^3-\left(-\frac{1}{3}(0)^3\right)+2(2)-2(0)+\frac{1}{2}(2)^2-\frac{1}{2}(0)^2$$

$$-8/3-0+4-0+2 = -8/3+6 = 10/3$$

$\frac{18}{3}$

(3)



$$\int_{-3}^{3}(9-x^2)\,dx = 2\int_{-\sqrt{k}}^{\sqrt{k}}(k-x^2)\,dx$$

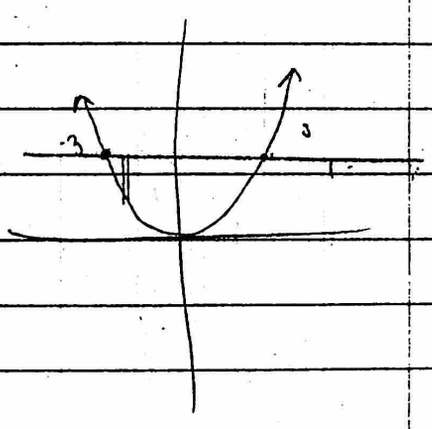
$$9x-\frac{1}{3}x^3\Big]_{-3}^{3} = 2\cdot\left(kx-\frac{1}{3}x^3\right]_{-\sqrt{k}}^{\sqrt{k}}$$

$x^2 = k$

$x = \sqrt{k}$

$$9(3)-\tfrac{1}{3}(3)^3-\left(9(-3)-\tfrac{1}{3}(-3)^3\right)$$

$$27-9+27-9$$



$$36 = 2\cdot\left[k\sqrt{k}-\tfrac{1}{3}(\sqrt{k})^3-\left(k(-\sqrt{k})-\tfrac{1}{3}(-\sqrt{k})^3\right)\right]$$

$$36 = 2\cdot\left(k^{3/2}-\tfrac{1}{3}k^{3/2}+k^{3/2}-\tfrac{1}{3}k^{3/2}\right)$$

$$36 = 2\left(2k^{3/2}-\tfrac{2}{3}k^{3/2}\right)$$

$$36 = 2\left(\tfrac{4}{3}k^{3/2}\right)$$

$$\tfrac{3}{4}\cdot 18 = \tfrac{4}{3}k^{3/2}\cdot\tfrac{3}{4}$$

$$\frac{54}{4} = k^{3/2}$$

$$\frac{27}{2} = k^{3/2}$$

$$k = \left(\tfrac{27}{2}\right)^{2/3} \approx 5.669644$$

$$5.670$$