Do Now:

| $x$ | 2 | 3 | 5 | 8 | 13 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 4 | –2 | 3 | 6 |

5. Let $f$ be a function that is twice differentiable for all real numbers. The table above gives values of $f$ for selected points in the closed interval $2 \le x \le 13$.

   (a) Estimate $f'(4)$. Show the work that leads to your answer.

   $f'(4) \approx \frac{f(5)-f(3)}{5-3} = \frac{-2-4}{2} = -3$

   (b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

   (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

   $(1)(1) + (2)(4) + (3)(-2) + (5)(3) = 18$

   (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all $x$ in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of $f$ at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of $f$ on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

b) $\int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$

$3x\Big|_2^{13} - 5\left[f(13) - f(2)\right]$

$3(13) - 3(2) - 5[6 - 1]$

$33 - 25 = 8$

d) $f'(5) = 3$ $\quad f(5) = -2$

$y + 2 = 3(x-5)$

$y + 2 = 3(7-5)$

$y + 2 = 6$

$y = 4$

Since $f''(x) < 0$ this would be an overapproximation

$\underline{\text{secant line}}$ $\frac{f(8) - f(5)}{8-5} = \frac{3-(-2)}{3} = \frac{5}{3}$

$(5, -2)$

$y + 2 = \frac{5}{3}(x-5)$

$y + 2 = \frac{5}{3}(7-5)$

$y + 2 = \frac{10}{3}$

$y = \frac{4}{3}$ ← this would be an underapproximation

Since $f''(x) < 0$

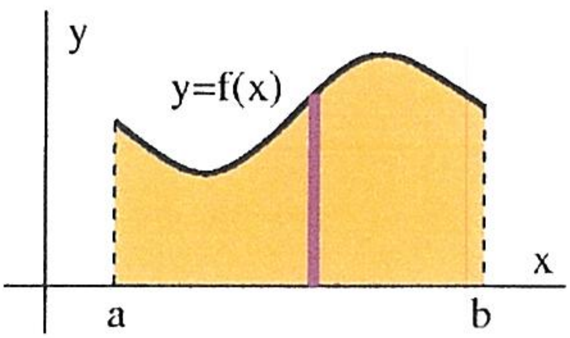


GO ON TO THE NEXT PAGE.

Name: ______________________________ Date:__________

AP Calculus AB: Area Between 2 Curves Ms. Loughran

Remember:




If a function $f$ is continuous on $[a,b]$ and if $f(x) \geq 0$ for all $x$ in $[a,b]$ then the area under the curve $y = f(x)$ over the interval $[a,b]$ is defined by:

$$Area = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

Which can be rewritten as : $Area = \int_a^b f(x)dx$

What if the region is not bounded by the $x$-axis?
What if the area is between 2 curves?

**Vertical Strip:** (everything in terms of x)

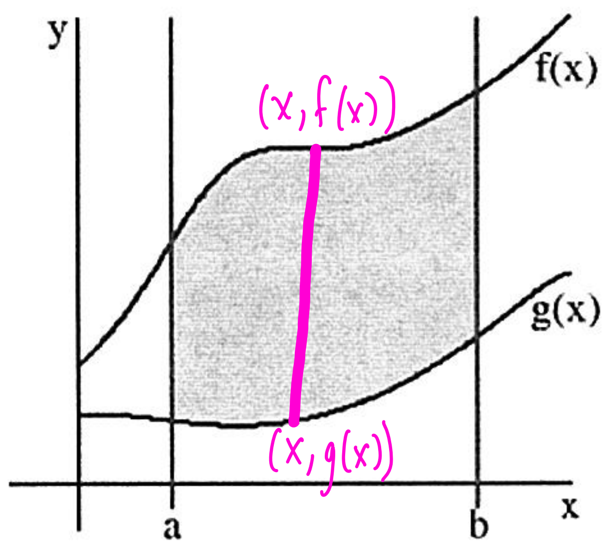
right most x-value

top curve

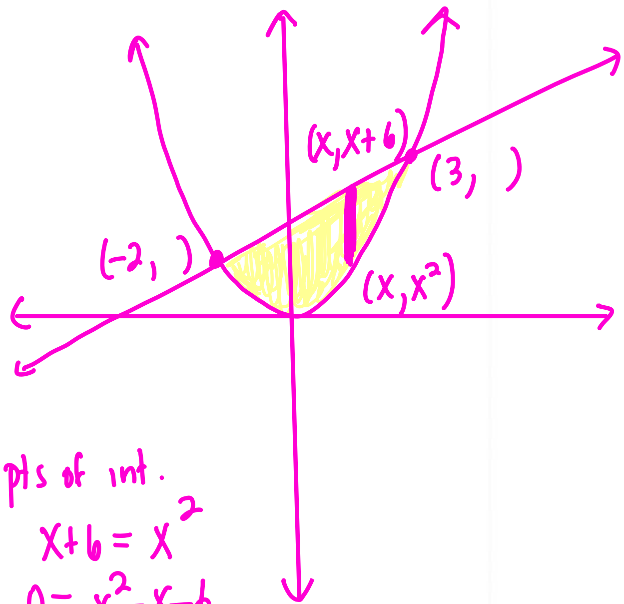
bottom curve

$$Area = \int_a^b (f(x) - g(x))dx$$

left most x-value



1. Find the area of the region bounded by $y = x + 6$ and $y = x^2$.




$$\int_{-2}^{3} \left( (x+6) - x^2 \right) dx$$

$$\left. \frac{x^2}{2} + 6x - \frac{x^3}{3} \right|_{-2}^{3} = \frac{125}{6}$$

pts of int.

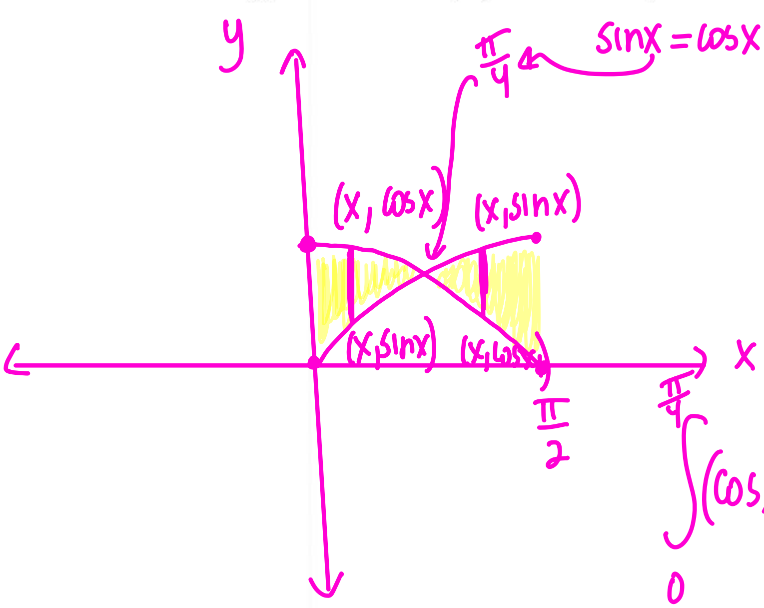
$x + 6 = x^2$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3, -2$

2. Find the area of the region bounded by $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$.




$$\int_{0}^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$$

$$\left. \sin x + \cos x \right|_{0}^{\frac{\pi}{4}} + \left. -\cos x - \sin x \right|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

Because of the symmetry you could do

$$2\int_{0}^{\frac{\pi}{4}} (\cos x - \sin x)\,dx \text{ or } 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$$

$$= 2\sqrt{2} - 2$$

pts of int.

$x^3-6x^2+8x=x^2-4x$

3. Find the area of the region bounded by $y = x^3 - 6x^2 + 8x$ and $y = x^2 - 4x$.

$x^3-7x^2+12x=0$

$x(x^2-7x+12)=0$

$x(x-4)(x-3)=0$

$x=0,4,3$

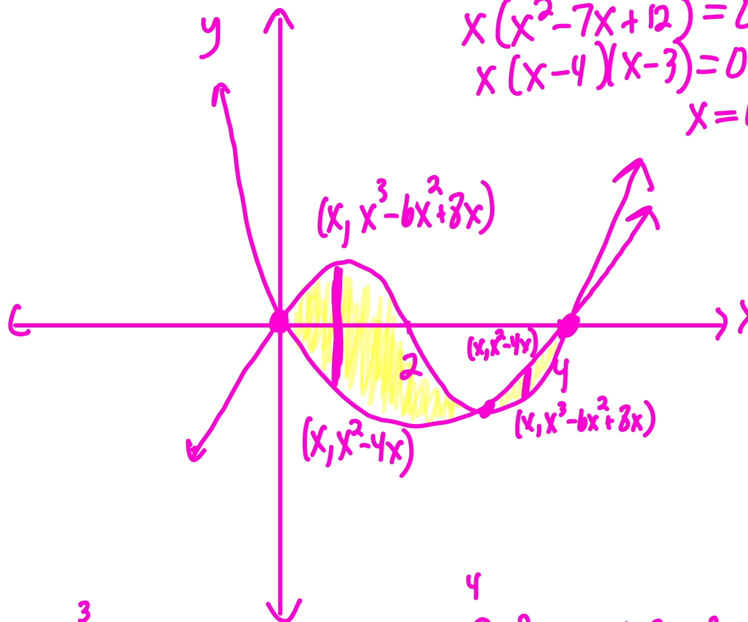
$y=x^3-6x^2+8x$

$0=x(x^2-6x+8)$

$0=x(x-4)(x-2)$

$x=0,4,2$

$-\quad 0 \quad + \quad 2 \quad - \quad 4 \quad + \quad y$




$y=x^2-4x$

$0=x(x-4)$

$x=0,4$

$+\quad 0 \quad - \quad 4 \quad +$

$$\int_0^3 x^3-6x^2+8x-(x^2-4x)\,dx+\int_3^4 x^2-4x-(x^3-6x^2+8x)\,dx$$

$$\int_0^3(x^3-7x^2+12x)\,dx+\int_3^4(7x^2-12x-x^3)\,dx$$

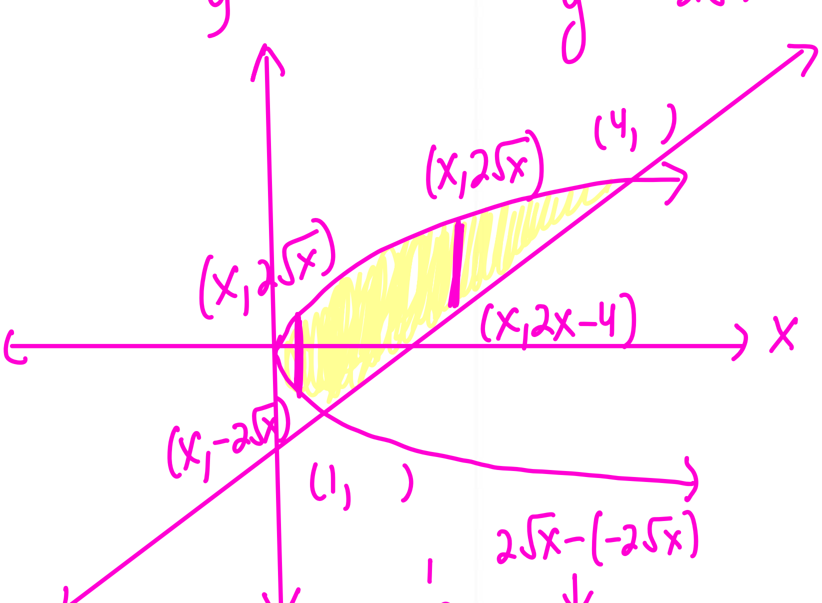
4. Find the area of the region enclosed by $y^2 = 4x$ and $y = 2x - 4$.

$$\left.\frac{x^4}{4}-\frac{7x^3}{3}+6x^2\right|_0^3+\left.\frac{7x^3}{3}-6x^2-\frac{x^4}{4}\right|_3^4$$

$=\frac{71}{6}$

$y=\pm\sqrt{4x}$

$y=\pm2\sqrt{x}$




pt of int.

$(2x-4)^2=4x$

$4x^2-16x+16=4x$

$4x^2-20x+16=0$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x=4,1$

$$\int_0^1\Big(\underbrace{2\sqrt{x}+2\sqrt{x}}_{2\sqrt{x}-(-2\sqrt{x})}\Big)dy+\int_1^4\big(2\sqrt{x}-(2x-4)\big)dx$$

$$\int_0^1 4\sqrt{x}\,dx+\int_1^4\left(2x^{\frac{1}{2}}-2x+4\right)dx=9$$