**Answers to today's classwork:**

a)

$(x,2)$, $y=2$

$(x,\sqrt{x})$, $y=\sqrt{x}$

$(x,0)$

$R = 2-0 = 2$

$r = \sqrt{x} - 0 = \sqrt{x}$

$$V = \pi \int_0^4 \left[(2)^2 - (\sqrt{x})^2\right] dx = \pi \int_0^4 (4-x)\,dx = \pi \cdot \left(4x - \frac{x^2}{2}\right)\Big]_0^4$$

$$\pi\left(4(4) - \frac{4^2}{2} - 0\right) = \boxed{8\pi}$$

b)

$y=\sqrt{x}$, $y=2$

$(0,y)$, $(y^2, y)$

$R = y^2 - 0$

$r = y^2$

$$V = \pi \int_0^2 (y^2)^2 \, dy = \pi \int_0^2 y^4 \, dy = \pi \cdot \frac{y^5}{5}\Big]_0^2$$

$$\pi\left(\frac{(2)^5}{5}\right) = \boxed{\frac{32\pi}{5}}$$

c)

$(x,2)$, $y=2$

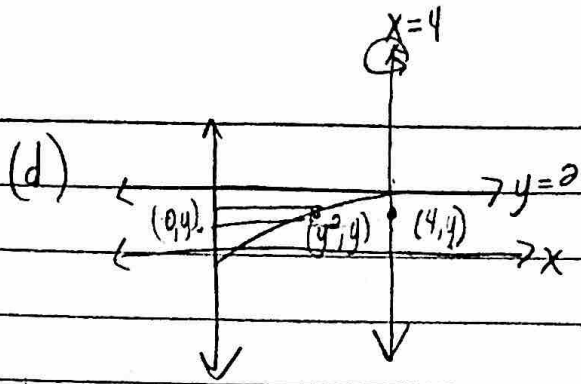$(x,\sqrt{x})$  $y=\sqrt{x}$

$r = 2 - \sqrt{x}$

$$V = \pi \int_0^4 (2-\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 \left(4 - 4\sqrt{x} + x\right) dx$$

$$V = \pi \cdot \left. 4x - \frac{8x^{3/2}}{3} + \frac{x^2}{2} \right|_0^4$$

$$\pi\left(\left(4(4) - \frac{8(4)^{3/2}}{3} + \frac{4^2}{2}\right) - 0\right) = \pi\left(16 - \frac{64}{3} + 8\right)$$

$$= \pi\left(24 - \frac{64}{3}\right) = \boxed{\frac{8\pi}{3}}$$

(d)

$x=4$

(0,4)  ($y^2$,y)  (4,4)  $y=2$

$x$

$R = 4-0 = 4$

$r = 4-y^2$

$V = \pi \int_0^2 \left[ (4)^2 - (4-y^2)^2 \right] dy$

$V = \pi \int_0^2 \left( 16 - (16 - 8y^2 + y^4) \right) dy$

$= \pi \int_0^2 \left( 8y^2 - y^4 \right) dy$

$\pi \cdot \frac{8}{3}y^3 - \frac{y^5}{5} \Big|_0^2 =$

$\pi \left( \frac{8}{3}(2)^3 - \frac{(2)^5}{5} \right) = \pi \left( \frac{64}{3} - \frac{32}{5} \right)$

$= \boxed{\frac{224\pi}{15}}$

(e)

$y$

(x,2)

(x,$\sqrt{x}$)  $y=\sqrt{x}$  $y=2$

$x$

(x,-1)

$R = 2-(-1) = 3$

$r = \sqrt{x} - (-1) = \sqrt{x} + 1$

$-\int 2\sqrt{x}$
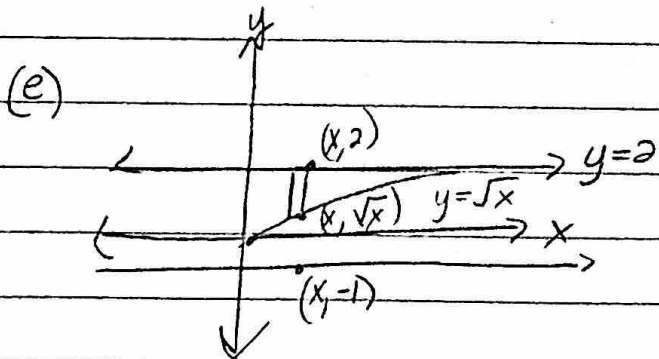
$-2\int x^{\frac{1}{2}}$

$-2 \cdot \frac{2}{3} x^{3/2}$

$-\frac{4}{3} x^{3/2}$

$V = \pi \int_0^4 \left[ (3)^2 - (\sqrt{x} + 1)^2 \right] dx$

$V = \pi \int_0^4 9 - \left( x + 2\sqrt{x} + 1 \right) dx$

$V = \pi \int_0^4 \left( 8 - x - 2\sqrt{x} \right) dx$

$V = \pi \cdot \left( 8x - \frac{x^2}{2} - \frac{4x^{3/2}}{3} \right) \Big|_0^4$

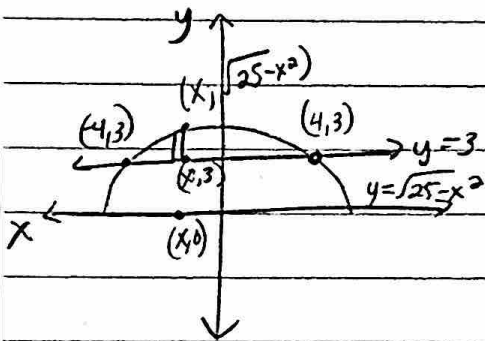$V = \pi \left( 8(4) - \frac{(4)^2}{2} - \frac{4(4)^{3/2}}{3} \right)$

$V = \pi \left( 32 - 8 - \frac{32}{3} \right) = \pi \left( 24 - \frac{32}{3} \right)$

$= \boxed{\frac{40\pi}{3}}$

⑨ $y = \sqrt{25 - x^2}$, $y = 3$

$x^2 + y^2 = 25$



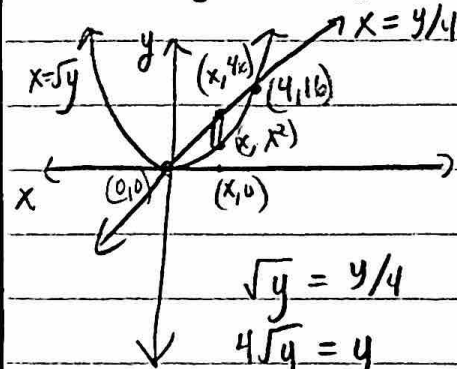$R = \sqrt{25 - x^2} - 0 = \sqrt{25 - x^2}$

$r = 3 - 0 = 3$

$V = \pi \int_{-4}^{4} \left( \left( \sqrt{25-x^2} \right)^2 - (3)^2 \right) dx$

$V = \pi \int_{-4}^{4} (25 - x^2 - 9) dx = \pi \int_{-4}^{4} 16 - x^2 = \pi \cdot \left[ 16x - \frac{x^3}{3} \right]_{-4}^{4}$

$\pi \left[ 16(4) - \frac{4^3}{3} - \left( 16(-4) - \frac{(-4)^3}{3} \right) \right]$

$\pi \left( 64 - \frac{64}{3} + 64 - \frac{64}{3} \right)$

$\pi \left( 128 - \frac{128}{3} \right) = \boxed{256\pi/3}$

$\rightarrow x^2 = y$    $4x = y$

⑬ $x = \sqrt{y}$    $x = y/4$



$\sqrt{y} = y/4$

$4\sqrt{y} = y$

$16y = y^2$

$y^2 - 16y = 0$

$y(y - 16) = 0$

$y = 0$ | $y = 16$

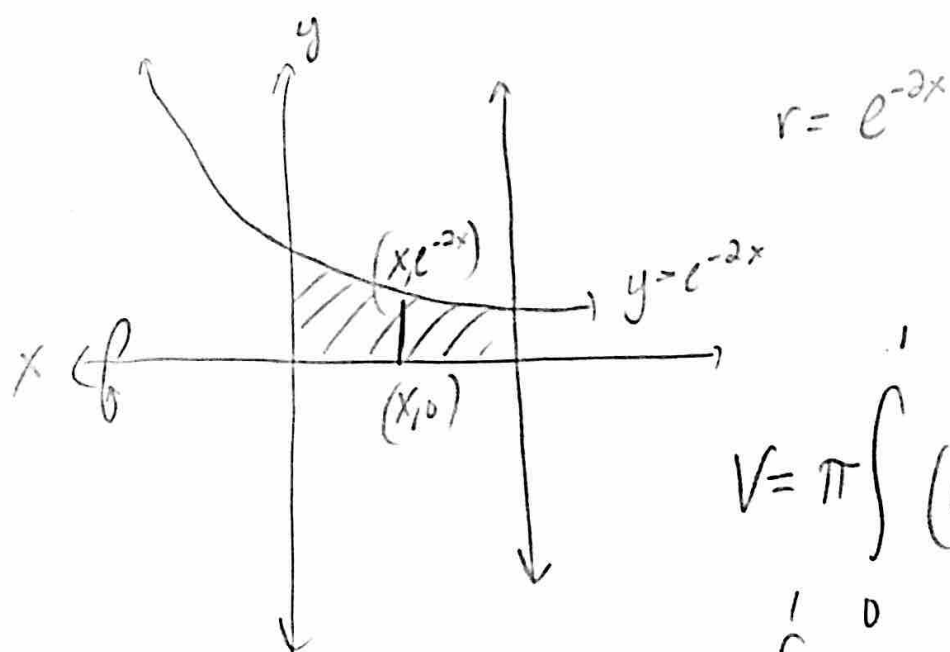$x = 0$    $x = 4$

$R = 4x - 0 = 4x$

$r = x^2 - 0 = x^2$

$V = \pi \int_{0}^{4} \left( (4x)^2 - (x^2)^2 \right) dx$

$V = \pi \int_{0}^{4} (16x^2 - x^4) dx = \pi \cdot \left[ \frac{16x^3}{3} - \frac{x^5}{5} \right]_{0}^{4}$

$= \pi \left[ \frac{16(4)^3}{3} - \frac{(4)^5}{5} - 0 \right] = \pi \left[ \frac{1024}{3} - \frac{1024}{5} \right]$

$\pi \left[ \frac{5120}{15} - \frac{3072}{15} \right] = \boxed{\frac{2048\pi}{15}}$

(12)



$r = e^{-2x}$

$y = e^{-2x}$

$(x, e^{-2x})$

$(x, 0)$

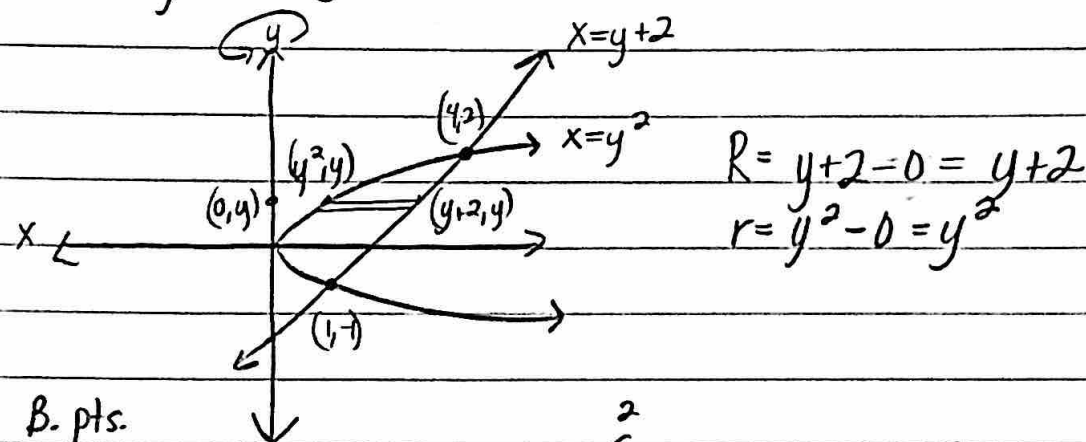$$V = \pi \int_0^1 \left(e^{-2x}\right)^2 dx$$

$$\pi \int_0^1 e^{-4x} dx =$$

$$= \pi \left[ -\frac{1}{4} e^{-4x} \Big|_0^1 \right] = \pi \left[ -\frac{1}{4} e^{-4} - \left( -\frac{1}{4} e^0 \right) \right] = \left( \frac{-1}{4e^4} + \frac{1}{4} \right) \pi$$

21) $x = y^2$, $x = y + 2$



$R = y + 2 - 0 = y + 2$

$r = y^2 - 0 = y^2$

B. pts.

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y - 2)(y + 1) = 0$

$y = 2 \quad y = -1$

$x = 4 \quad x = 1$

$$V = \pi \int_{-1}^{2} \left[ (y+2)^2 - (y^2)^2 \right] dy$$

$$= \pi \int_{-1}^{2} \left[ y^2 + 4y + 4 - y^4 \right] dy$$

$$V = \pi \left[ \frac{y^3}{3} + 2y^2 + 4y - \frac{y^5}{5} \Big|_{-1}^{2} \right]$$

$$V = \pi \left( \frac{(2)^3}{3} + 2(2)^2 + 4(2) - \frac{(2)^5}{5} - \left( \frac{(-1)^3}{3} + 2(-1)^2 + 4(-1) - \frac{(-1)^5}{5} \right) \right)$$

$$V = \pi \left( \frac{8}{3} + 8 + 8 - \frac{32}{5} - \left( \frac{-1}{3} + 2 - 4 + \frac{1}{5} \right) \right)$$

$$V = \pi \left( \frac{8}{3} + 16 - \frac{32}{5} + \frac{1}{3} - 2 + 4 - \frac{1}{5} \right)$$

$$V = \pi \left( 18 + 3 - \frac{33}{5} \right)$$

$$V = \pi \left( 21 - \frac{33}{5} \right) = \pi \left( \frac{105}{5} - \frac{33}{5} \right)$$

$$= \frac{72\pi}{5}$$