Name: ______________________________________ Date: _________
AP Calc Ms. Loughran

$\frac{0}{0}$ L'Hopitals

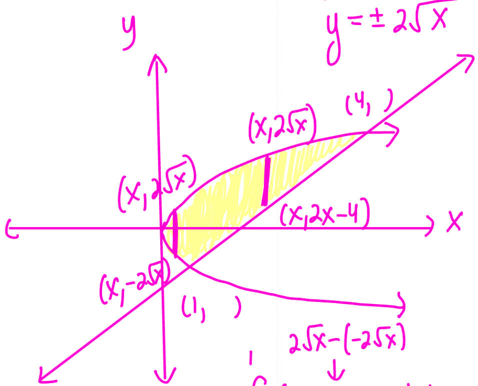
1. $\lim_{h \to 0} \frac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$ is $= \lim_{h \to 0} \frac{\sqrt{(1+h)^5+8}}{1} = \sqrt{9}$

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) nonexistent

4. Find the area of the region enclosed by $y^2 = 4x$ and $y = 2x - 4$.

$y = \pm\sqrt{4x}$

$y = \pm 2\sqrt{x}$



pt of int.

$(2x-4)^2 = 4x$

$4x^2 - 16x + 16 = 4x$

$4x^2 - 20x + 16 = 0$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4, 1$

$2\sqrt{x} - (-2\sqrt{x})$

$$\int_0^1 \left(2\sqrt{x} + 2\sqrt{x}\right)dx + \int_1^4 \left(2\sqrt{x} - (2x-4)\right)dx$$

$$\int_0^1 4\sqrt{x}\,dx + \int_1^4 \left(2\sqrt{x} - 2x + 4\right)dx = 9$$

OR

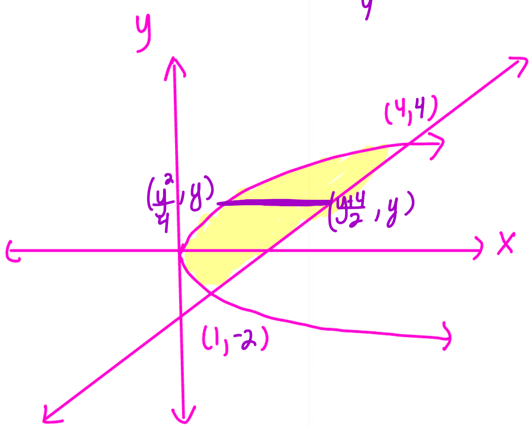
4. Find the area of the region enclosed by $y^2 = 4x$ and $y = 2x - 4$.

$y^2 = 4x$

$\frac{y^2}{4} = x$

$y = 2x - 4$

$\frac{y+4}{2} = x$



$$\int_{-2}^{4} \left(\frac{y+4}{2} - \frac{y^2}{4}\right)dy = 9$$

Horizontal Strip:

(everything in terms of y)

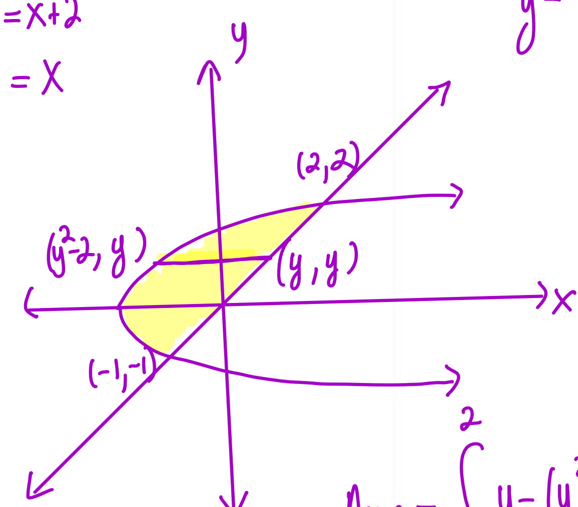
$$A = \int_{\text{bottom most } y\text{-value}}^{\text{top most } y\text{ value}} (\text{right most curve} - \text{left most curve})\, dy$$

5. Find the area of the region enclosed by $y^2 = x+2$ and $y = x$.

$y^2 = x+2$

$y^2 - 2 = x$

$y = \pm\sqrt{x+2}$



pts of int.

$x^2 = x+2$

$x^2 - x - 2 = 0$

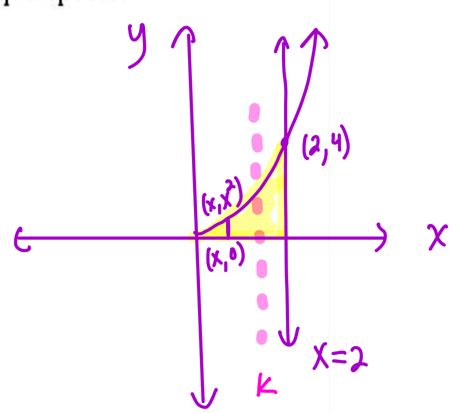
$(x-2)(x+1) = 0$

$x = 2, -1$

$$\text{Area} = \int_{-1}^{2} y - (y^2 - 2)\, dy = \int_{-1}^{2} (y - y^2 + 2)\, dy = \frac{y^2}{2} - \frac{y^3}{3} + 2y \Big|_{-1}^{2} = \frac{9}{2}$$

6. Find a vertical line $x = k$ that divides the area enclosed by $x = \sqrt{y}$, $x = 2$ and $y = 0$ into 2 equal parts.

$x^2 = y$



$$\int_0^2 (x^2 - 0)\, dx = 2\int_0^k (x^2 - 0)\, dx$$

$$\int_0^2 x^2 dx = 2\int_0^k x^2 dx$$

$$\frac{x^3}{3}\Big|_0^2 = 2 \cdot \frac{x^3}{3}\Big|_0^k$$

$$\frac{8}{3} = \frac{2k^3}{3}$$

$2k^3 = 8$

$k^3 = 4$

$k = \sqrt[3]{4}$

Homework 03-14

① $\int_{-1}^{2}\left((x^2+1)-x\right)dx = \int_{-1}^{2}(x^2+1)dx - \int_{-1}^{2}x\,dx$

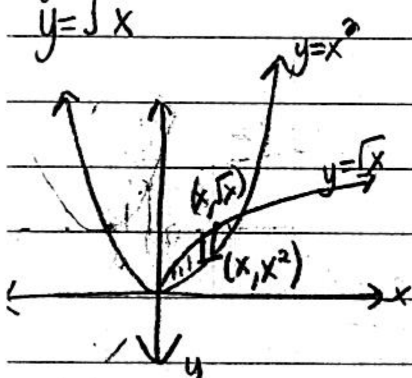
$\frac{1}{3}x^3+x\Big]_{-1}^{2} - \frac{1}{2}x^2\Big]_{-1}^{2} = \left(\frac{1}{3}(2)^3+2\right)-\left(\frac{1}{3}(-1)^3-1\right)-\left[\left(\frac{1}{2}(2)^2\right)-\frac{1}{2}(-1)^2\right]$

$\frac{14}{3}+\frac{4}{3}-\left(2-\frac{1}{2}\right)$

$\frac{18}{3}-\frac{3}{2} = 6-\frac{3}{2} = \frac{9}{2}$

②
$y=x^2 \qquad x=\frac{1}{4},\ x=1$

$y=\sqrt{x}$



$\int_{\frac{1}{4}}^{1}\left(\sqrt{x}-x^2\right)dx = \frac{2}{3}x^{3/2}\Big]_{\frac{1}{4}}^{1} - \frac{1}{3}x^3\Big]_{\frac{1}{4}}^{1}$

$\frac{2}{3}(1)^{3/2}-\frac{2}{3}\left(\frac{1}{4}\right)^{3/2}-\left(\frac{1}{3}(1)^3-\frac{1}{3}\left(\frac{1}{4}\right)^3\right)$

$\frac{2}{3}-\frac{1}{12}-\left(\frac{1}{3}-\frac{1}{192}\right)$

$\frac{8}{12}-\frac{1}{12}-\left(\frac{64}{192}-\frac{1}{192}\right)$

$\frac{(16)}{(16)}\,7/12 - 63/192 = \frac{112-63}{192} = \frac{49}{192}$

oundary pts

$x^2=\sqrt{x}$

$x^4=x$

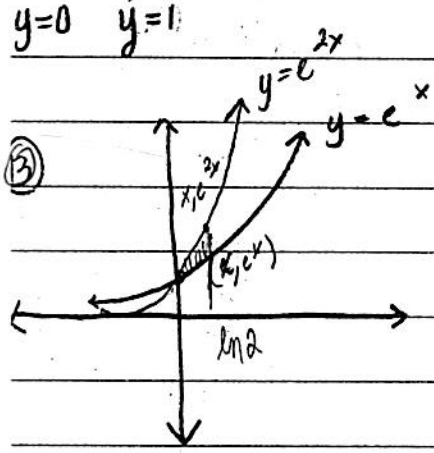
$x^4-x=0$

$x\ |\ x^3-1=0$

$x=0\ |\ x=1$

$y=0 \quad y=1$

③



$\int_{0}^{\ln 2}\left(e^{2x}-e^{x}\right)dx = \int_{0}^{\ln 2}e^{2x}dx - \int_{0}^{\ln 2}e^{x}dx$

$u=2x$

$du=2dx$

$\frac{du}{2}=dx$

$\frac{1}{2}e^{2x}\Big]_{0}^{\ln 2} - e^{x}\Big]_{0}^{\ln 2}$

$\frac{1}{2}e^{2(\ln 2)}-\frac{1}{2}e^{2(0)}-\left(e^{\ln 2}-e^{0}\right) = \frac{1}{2}(4)-\frac{1}{2}-(2-1)$

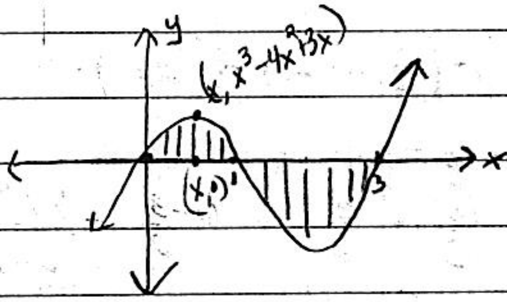
$2-\frac{1}{2}-1=\frac{1}{2}$

(H) $y = x^3 - 4x^2 + 3x$, $y=0$, $x=0$, $x=3$

$x(x^2-4x+3)$

$x(x-1)(x-3)$

y: $-$ 0 $+$ 1 $-$ 3 $+$



$$\int_0^1 (x^3 - 4x^2 + 3x - 0)\,dx + \int_1^3 0 - (x^3 - 4x^2 + 3x)\,dx$$

$37/12 \approx 3.08\overline{3}$