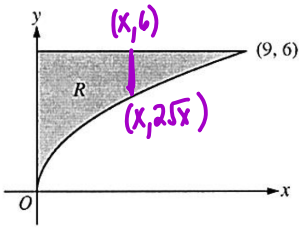
Do Now:

2010 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

   (a) Find the area of R.

$$\text{Area} = \int_0^9 (6 - 2\sqrt{x})\,dx$$

$$= 6x - \frac{4}{3}x^{3/2}\Big|_0^9$$

$$6(9) - \frac{4}{3}(9)^{3/2}$$

$$54 - \frac{108}{3} - 0 = 54 - \frac{108}{3}$$

2001 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

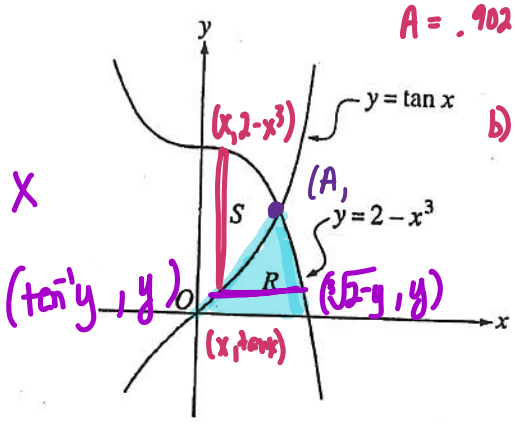
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

$y = 2 - x^3$

b) $\sqrt[3]{2-y} = x$

$A = .9021\ldots$



b) $\text{Area}_S = \int_0^A \left((2-x^3) - \tan x\right) dx$

$= 1.161$ or 1.160

1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.
   (a) Find the area of R.
   (b) Find the area of S.
   (c) Find the volume of the solid generated when S is revolved about the x-axis.

$\tan A$ or $2 - A^3$

a) $\int_0^{\text{A}} \left(\sqrt[3]{2-y} - \tan^{-1} y\right) dy = .729$

Name: _____ Date: _____

AP Calc: Writing Limits of Riemann Sums as Definite Integrals

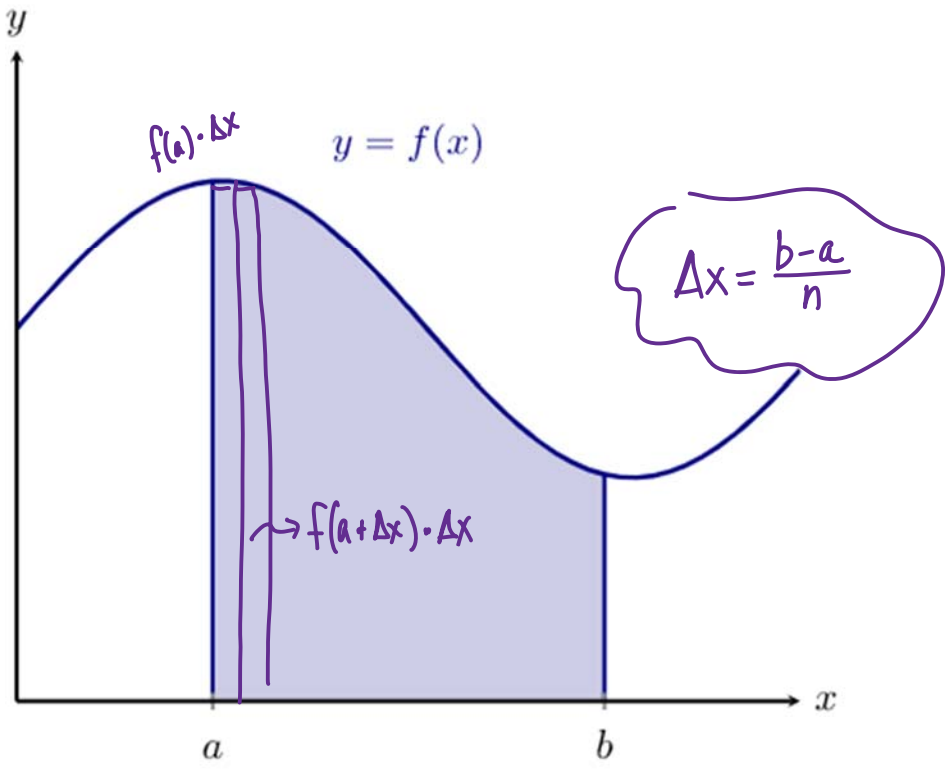
Recall:

If a function f is continuous on $[a,b]$ and if $f(x) \geq 0$ for all x in $[a,b]$ then the area under the curve $y = f(x)$ over the interval $[a,b]$ is defined by:

$$Area = \lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

height

base of rectangles



Which can be rewritten as : $$Area = \int_{a}^{b} f(x)dx$$

Δx

f(a+ΔxK)

$f(a+\Delta x k)$

1. Given $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3\left(3+\frac{2k}{n}\right)+1\right)\cdot\frac{2}{n}$, write as an equivalent definite integral.

Δx

$a+\Delta x k$

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$

$f(x) = 3x+1$

$b-a = 2$

$a = 3$

$b-3 = 2$

$b = 5$

$\int_3^5 (3x+1)\,dx$

2. Given $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\frac{5k}{n}\right)^2+2\right)\cdot\frac{5}{n}$, write as an equivalent definite integral.

$\Delta x = \frac{5}{n} = \frac{b-a}{n}$

$a+\Delta x k$

$a = 0$

$5 = b-a$

$5 = b-0$

$5 = b$

$f(x) = x^2+2$

$\int_0^5 (x^2+2)\,dx$

3. Given $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{4k}{n}\right)^2\cdot\frac{4}{n}$, write as an equivalent definite integral.

$\Delta x = \frac{4}{n} = \frac{b-a}{n}$

$a+\Delta x k$

$a = 3$

$4 = b-a$

$4 = b-3$

$7 = b$

$f(x) = x^2$

$\int_3^7 x^2\,dx$

Now what if we have to go in the reverse?

4. Given $\int_0^3 e^x dx$, write it as an equivalent limit of a Riemann sum

$f(a+\Delta x\, k)\cdot \Delta x$

$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$

$\lim_{n\to\infty} \sum_{k=1}^{n} e^{\frac{3}{n}k} \cdot \frac{3}{n}$

$f(x) = e^x$

$f(a+\Delta x\, k)$

$f(0+\frac{3}{n}k) = e^{\frac{3}{n}k}$

WHO?

Which of the limits is equivalent to the following definite integral?

$$\int_0^{\pi} \cos x \, dx$$

(A) $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{i}{n}\right) \cdot \frac{\pi}{n}$ STACK

(B) $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi i}{n}\right) \cdot \frac{i}{n}$ LEE

(C) $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi i}{n}\right) \cdot \frac{\pi}{n}$ CARMAN

(D) $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{i}{n}\right) \cdot \frac{i}{n}$ LOUGHRAN

WHEN?

Which of the limits is equivalent to the following definite integral?

$$\int_{-2}^{3} (x+1)\,dx$$

(A) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n} - 1\right) \cdot \frac{5}{n}$	PERIOD 1
(B) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i-1}{n}\right) \cdot \frac{5}{n}$	PERIOD 3
(C) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i+1}{n} + 1\right) \cdot \frac{5}{n}$	PERIOD 5
(D) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n} + 1\right) \cdot \frac{5}{n}$	PERIOD 7

WHAT? (PART 1)

Which of the definite integrals is equivalent to the following limit?

$$\lim_{n\to\infty}\sum_{i=1}^{n}\ln\left(2+\frac{5i}{n}\right)\cdot\frac{5}{n}$$

(A) $\int_0^7 \ln x\,dx$ ICE CREAM

(B) $\int_2^5 \ln x\,dx$ CUPCAKES

(C) $\int_2^7 \ln x\,dx$ REESES PB CUPS

(D) $\int_0^5 \ln x\,dx$ PEPPERMINT PATTIES

WHAT? (PART 2)

Which of the definite integrals is equivalent to the following limit?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi}{2} + \frac{\pi i}{2n}\right) \cdot \frac{\pi}{2n}$$

Ⓐ $\int_0^{\pi} \cos x\, dx$ GREEN TEA

Ⓑ $\int_{\pi/2}^{3\pi/4} \cos x\, dx$ ICED CARAMEL LATTE

Ⓒ $\int_0^{\pi/2} \cos x\, dx$ PEACH SELTZER

Ⓓ $\int_{\pi/2}^{\pi} \cos x\, dx$ HOT CHOCOLATE

WITH?

Which of the definite integrals is equivalent to the following limit?

$$\lim_{n\to\infty} \sum_{i=1}^{n} 4 \cdot \frac{5}{n}$$

(A) $\int_0^4 5x\,dx$	PAPER CLIP
(B) $\int_0^5 4x\,dx$	SCREWDRIVER
(C) $\int_0^4 5\,dx$	HAMMER
(D) $\int_0^5 4\,dx$	COMPASS

WHERE?

Which of the definite integrals is equivalent to the following limit?

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{4+\frac{5i}{n}}\cdot\frac{5}{n}$$

(A) $\int_0^4 \sqrt{4+x}\,dx$ LIBRARY

(B) $\int_0^5 \sqrt{x}\,dx$ MAIN OFFICE

(C) $\int_4^9 \sqrt{x}\,dx$ TESTING CENTER

(D) $\int_4^9 \sqrt{4+x}\,dx$ SCIENCE STUDY CENTER

Homework 03-14

2002 Form B

1a) 3.215 or 3.214

2003

1a) .443 or .442

2003 Form B

$f'(x) = 8x - 3x^2$

1a) $f'(3) = 8(3) - 3(3)^2 = -3$

if $x=3$, $y = 4(3)^2 - 3(3)^3 = 9$

$y - 9 = -3(x-3)$

$y = -3x + 18$

1b) 7.917 or 7.916

2004

2a) 1.133

2004 Form B

1a) 18