Do Now:

2012 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

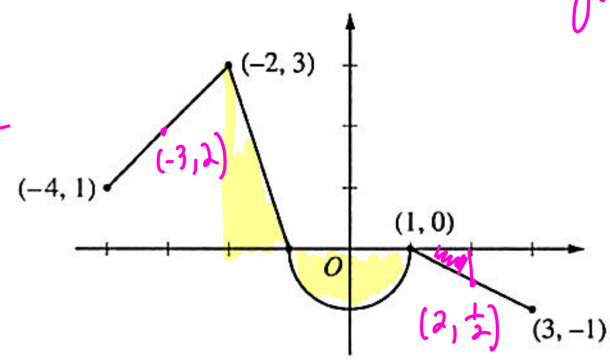
a) $g(2) = \int_1^2 f(t)\,dt$

$= -\frac{1}{2}(1)(\frac{1}{2}) = -\frac{1}{4}$

$g(-2) = -\int_{-2}^{1} f(t)\,dt$

$-\left(\frac{1}{2}(1)(3) - \frac{1}{2}\pi(1)^2\right)$

$-\left(\frac{3}{2} - \frac{\pi}{2}\right)$



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

$g'(x) = f(x)$

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$g'(-3) = f(-3) = 2$

$g''(-3) = 1 \leftarrow$ slope of g' at $x = -3$

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

g'' + − or − + $x = -2, 0, 1$

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g' ↗ ↘ or ↘ ↗   At these values g' changes from ↗ to ↘ or ↘ to ↗.

c) $g'(x) = 0$ at $x = \pm 1$

at $x = -1$ there is a rel max b/c g' changes sign from + to − at $x = -1$

$x = 1$ there is neither b/c g' doesn't change sign

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

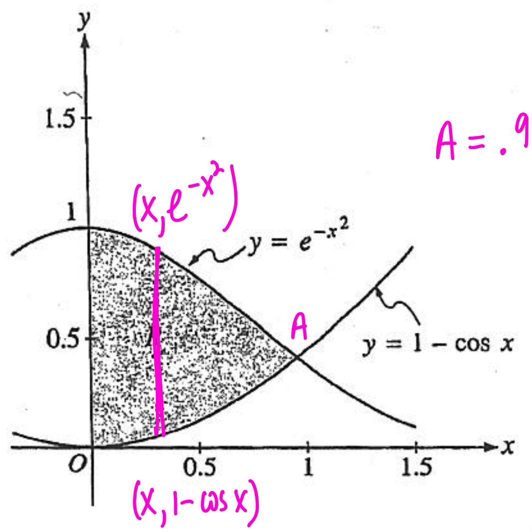
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



$A = .94194...$

$$A = \int_0^A \left(e^{-x^2} - (1-\cos x)\right) dx$$

$= .591$ or $.590$

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

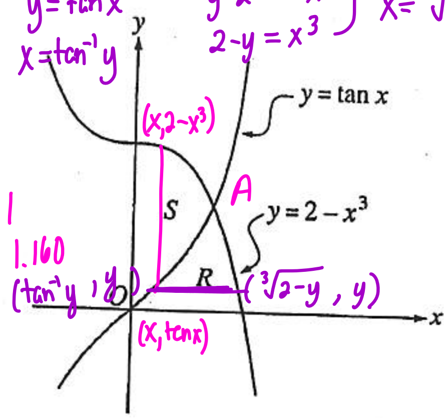
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

$y = 2-x^3$
$y-2 = -x^3$
$2-y = x^3$
$x = \sqrt[3]{2-y}$

$y = \tan x$
$x = \tan^{-1} y$

A = .9021...



b) $A_S = \int_0^A (2-x^3-\tan x)\,dx = 1.161$ or 1.160

a) $A_R = \int_0^{2-A^3 \text{ or } \tan A} \left(\sqrt[3]{2-y} - \tan^{-1} y\right) dy$

$= .729$

1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

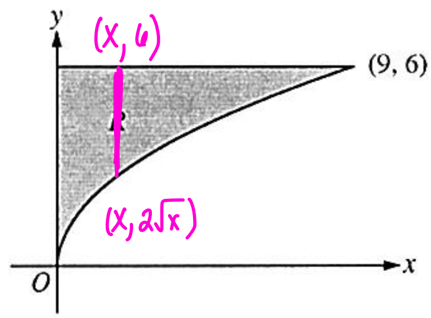
   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the x-axis.

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



a) $A_R = \int_0^9 (6-2\sqrt{x})\,dx$

$\left(6x - 2\cdot\frac{2}{3}x^{3/2}\right)\Big|_0^9$

$\left(6x - \frac{4}{3}x^{3/2}\right)\Big|_0^9 = 6(9) - \frac{4}{3}(9)^{3/2} - 0$

$6(9) - \frac{4}{3}(27)$

4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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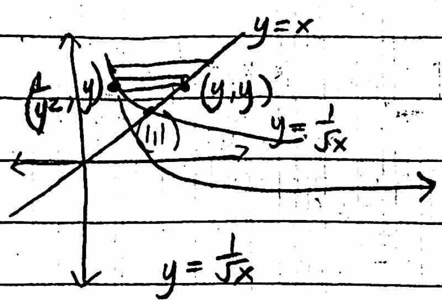
1) $y=x,\quad y=\frac{1}{\sqrt{x}}$



$x=\frac{1}{\sqrt{x}}$

$x\sqrt{x}=1$

$x^3=1$

$x=1$

$(1,1)$

$y=\frac{1}{\sqrt{x}}$

$\sqrt{x}\,y=1$

$\sqrt{x}=\frac{1}{y}$

$x=\frac{1}{y^2}$

$\frac{1}{\sqrt{x}}=2$

$2\sqrt{x}=1$

$\sqrt{x}=\frac{1}{2}$

$x=\frac{1}{4}$

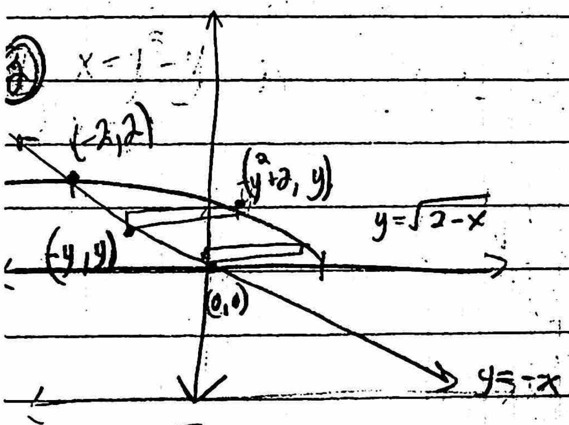
y^{-2}

$$\int_1^2 \left(y-\frac{1}{y^2}\right)dy = \frac{1}{2}y^2\Big]_1^2 - \left(-\frac{1}{y}\right]_1^2$$

$$\frac{1}{2}(2)^2-\frac{1}{2}(1)^2-\left(-\frac{1}{2}-(-1)\right)$$

$$2-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-1 = 1$$

2) $x=1-$



$-x=\sqrt{2-x}$

$x^2=2-x$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2\quad x=1$

$y=2\quad y=-1$

$y=\sqrt{2-x}$

$y^2=2-x$

$y^2-2=-x$

$-y^2+2=x$

$y=-x$

$-y=x$

$$\int_0^2 -y^2+2-(-y)\,dy$$

$$\int_0^2 -y^2+2+y\,dy$$

$$\int_0^2 -y^2dy+\int_0^2 2\,dy+\int_0^2 y\,dy$$

$$-\frac{1}{3}y^3\Big]_0^2+2y\Big]_0^2+\frac{1}{2}y^2\Big]_0^2$$

$$-\frac{1}{3}(2)^3-\left(-\frac{1}{3}(0)^3\right)+2(2)-2(0)+\frac{1}{2}(2)^2-\frac{1}{2}(0)^2$$

$$-8/3-0+4-0+2 = -8/3+6 = 10/3$$

$\frac{18}{3}$

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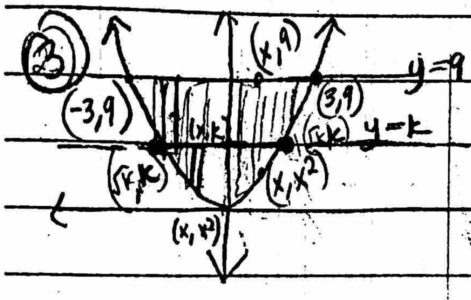

$$\int_{-3}^{3}\left(9-x^2\right)dx = 2\int_{-\sqrt{k}}^{\sqrt{k}}\left(k-x^2\right)dx$$

$$9x-\frac{1}{3}x^3\Big]_{-3}^{3} = 2\cdot\ kx-\frac{1}{3}x^3\Big]_{-\sqrt{k}}^{\sqrt{k}}$$

$x^2 = k$

$x = \sqrt{k}$

$$9(3)-\frac{1}{3}(3)^3-\left(9(-3)-\frac{1}{3}(-3)^3\right)$$

$$27-9+27-9$$

$$36 = 2\cdot\left[k\sqrt{k}-\frac{1}{3}(\sqrt{k})^3-\left(k(-\sqrt{k})-\frac{1}{3}(-\sqrt{k})^3\right)\right]$$

$$36 = 2\cdot\left(k^{3/2}-\frac{1}{3}k^{3/2}+k^{3/2}-\frac{1}{3}k^{3/2}\right)$$

$$36 = 2\left(2k^{3/2}-\frac{2}{3}k^{3/2}\right)$$

$$36 = 2\left(\frac{4}{3}k^{3/2}\right)$$

$$\frac{3}{4}\cdot 18 = \frac{4}{3}k^{3/2}\cdot\frac{3}{4}$$

$$\frac{54}{4} = k^{3/2}$$

$$\frac{27}{2} = k^{3/2}$$

$$k = \left(\frac{27}{2}\right)^{2/3} \approx 5.669644$$

$$5.670$$