Do Now:

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

   (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.

   $\int_0^5 E(t)\,dt = 153.457\ldots$ 153 fish

   (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?

   $\frac{1}{5-0}\int_0^5 L(t)\,dt = 6.059\ldots$ 6.059

   (c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

   (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$) ? Explain your reasoning.

(d) $E'(5) - L'(5) = -10.7227\ldots$

Since $E'(5) - L'(5) < 0$, the rate of change in the number of fish in the lake at 5am is decreasing.

$t = 6.203\ldots$

(c) $E(t) - L(t) = 0$

Since $E(t) - L(t) > 0$ prior to $t = 6.203$ and $E(t) - L(t) < 0$ after $t = 6.203$

Then at $t = 6.203$ we have a maximum

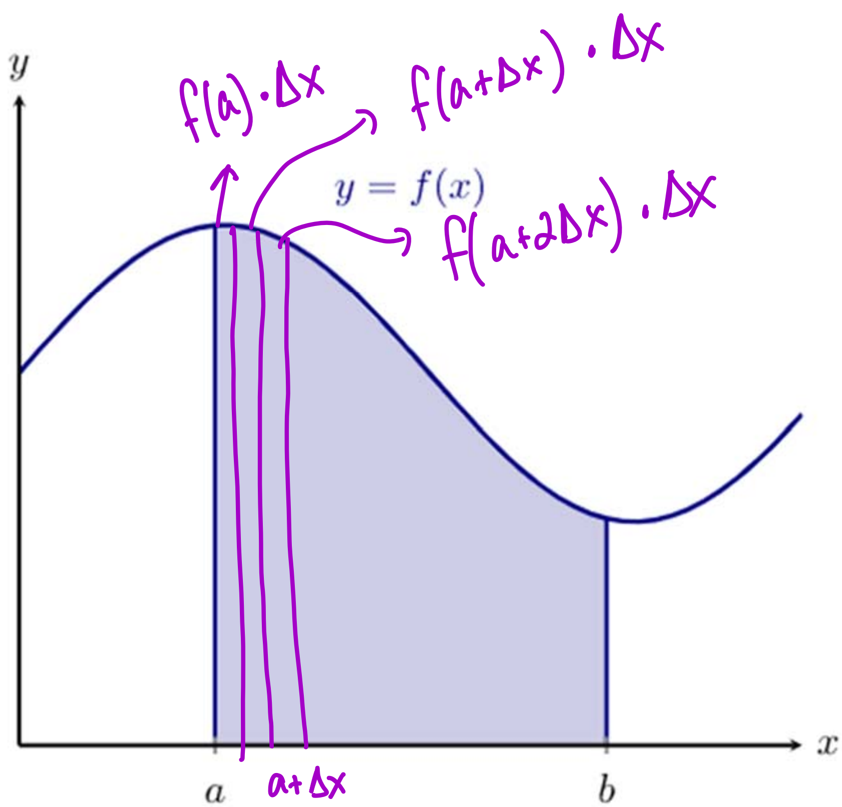
Name: ________________________________ Date: __________

AP Calc: Writing Limits of Riemann Sums as Definite Integrals

Recall:

If a function f is continuous on $[a,b]$ and if $f(x) \geq 0$ for all x in $[a,b]$ then the area under the curve $y = f(x)$ over the interval $[a,b]$ is defined by:

$$Area = \lim_{n \to +\infty} \sum_{k=1}^{n} \underbrace{f(x_k)}_{\text{hight}} \Delta x \rightarrow \text{base}$$



$$\Delta x = \frac{b-a}{n}$$

Which can be rewritten as : $Area = \int_a^b f(x)\,dx \rightarrow \Delta x$

$$f(x) \rightarrow f(a + \Delta x k)$$

$f(a+\Delta x k)$

1. Given $\lim_{n\to\infty}\sum_{k=1}^{n}\underbrace{\left(3\left(3+\frac{2k}{n}\right)+1\right)}_{}\left(\frac{2}{n}\right)$, write as an equivalent definite integral.

$\frac{2}{n} \to \Delta x$

$f(x) = 3x+1$

$\Delta x = \frac{2}{n}$

$3+\frac{2k}{n} = a+\Delta x k$

$a=3$

$\frac{b-a}{\cancel{n}} = \frac{2}{\cancel{n}}$

$b-3=2$

$b=5$

$\int_3^5 (3x+1)\,dx$

2. Given $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\left(\frac{5k}{n}\right)^2+2\right)\left(\frac{5}{n}\right)$, write as an equivalent definite integral.

$\frac{5k}{n} \to \Delta x k$

$\frac{5}{n} \to \Delta x$

$f(x) = x^2+2$

$\Delta x = \frac{5}{n}$

$a+\Delta x k = \frac{5k}{n}$

$\frac{5}{\cancel{n}} = \frac{b-a}{\cancel{n}}$

$a=0$

$5=b-a$

$5=b$

$\int_0^5 x^2+2\,dx$

3. Given $\lim_{n\to\infty}\sum_{k=1}^{n}\left(3+\frac{4k}{n}\right)^2\left(\frac{4}{n}\right)$, write as an equivalent definite integral.

$a+\Delta x$, Δx

$f(x) = x^2$

$a+\Delta x k = 3+\frac{4k}{n}$

$\frac{4}{\cancel{n}} = \frac{b-3}{\cancel{n}}$

$a=3$

$7=b$

$\int_3^7 x^2\,dx$

Now what if we have to go in the reverse?

4. Given $\int_0^3 e^x dx$, write it as an equivalent limit of a Riemann sum

$a=0 \quad b=3$

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{3-0}{n} = \frac{3}{n}$

$f(x) = e^x$

$f(a+\Delta x k)$

$f(0+\frac{3}{n}k) = f(\frac{3}{n}k) = e^{\frac{3}{n}k}$

$$\lim_{n\to\infty} \sum_{k=1}^{n} e^{\frac{3}{n}k} \frac{3}{n}$$

Homework 03-18

2002 Form B

1a) 3.215 or 3.214

2003

1a) .443 or .442

2004

2a) 1.133

2003 Form B

1a) $f'(x) = 8x - 3x^2$

$f'(3) = 8(3) - 3(3)^2 = -3$

if $x = 3$,

$y = 4(3)^2 - 3(3)^3 = 9$

$y - 9 = -3(x-3)$

$y = -3x + 18$

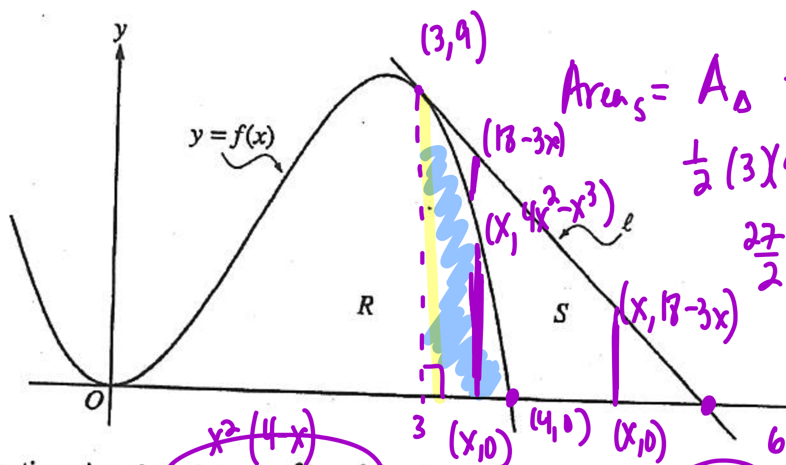
1b) 7.917 or 7.916

2004 Form B

1a) 18

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



$\text{Area}_S = A_\Delta - \int_3^4 4x^2 - x^2\,dx$

$\frac{1}{2}(3)(9)\ 3$

$\frac{27}{2}\ -$

$18-3x=0$
$18=3x$
$6=x$

$x^2(4-x)$

1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

or

$$A_s = \int_3^4 \left(18 - 3x - (4x^2 - x^3)\right)dx + \int_4^6 (18 - 3x - 0)\,dx$$