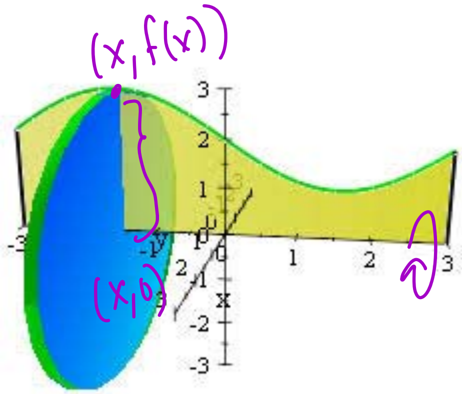
Definite Integral as the Limit of a Riemann Sum

$f(a+\Delta x k)\cdot \Delta x$

$\int_0^4 x^3\,dx$ $f(x)=x^3$ $a=0$ $f(0+\frac{4k}{n})=(\frac{4k}{n})^3$ $\Delta x=\frac{b-a}{n}=\frac{4-0}{n}=\frac{4}{n}$	Do Now: Fill in the table $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{4k}{n}\right)^3\cdot\frac{4}{n}$
$\int_2^4 x^3\,dx$	$f(a+\Delta x i)$ $\lim_{n\to\infty}\sum_{i=1}^{n}\left(2+\frac{2i}{n}\right)^3\cdot\left(\frac{2}{n}\right)$ Δx $\Delta x=\frac{b-a}{n}$ $a=2$ $\frac{2}{n}=\frac{b-2}{n}$ $4=b$
$\int_2^6 x^3\,dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left(2+\frac{4i}{n}\right)^3\cdot\frac{4}{n}$
$\int_4^6 x^3\,dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left(4+\frac{2i}{n}\right)^3\cdot\left(\frac{2}{n}\right)$
$(x+2)^3$ $\int_2^6 (x+2)^3\,dx$ $f(2+\frac{4}{n}i)$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left(4+\frac{4i}{n}\right)^3\left(\frac{4}{n}\right)$
$\int_0^{\pi}\sin x\,dx$	$\lim_{n\to\infty}\sum_{i=1}^{n}\left[\sin\left(\frac{\pi i}{n}\right)\right]\cdot\left(\frac{\pi}{n}\right)$

Name: ________________________________ Date: __________

AP Calculus: Volumes of Solids of Revolution Ms. Loughran



Vertical Strip:

$$\pi \int_{\text{left most x-value}}^{\text{right most x-value}} r^2 \, dx$$

r = top curve − bottom curve

Volume of each disk: $\pi r^2 h$ as $\Delta x \to 0 \to dx$

$r = f(x) - 0$

$r = f(x)$

r = top curve − bottom curve

$$\pi (f(x))^2 \Delta x$$

Volume of Solid:

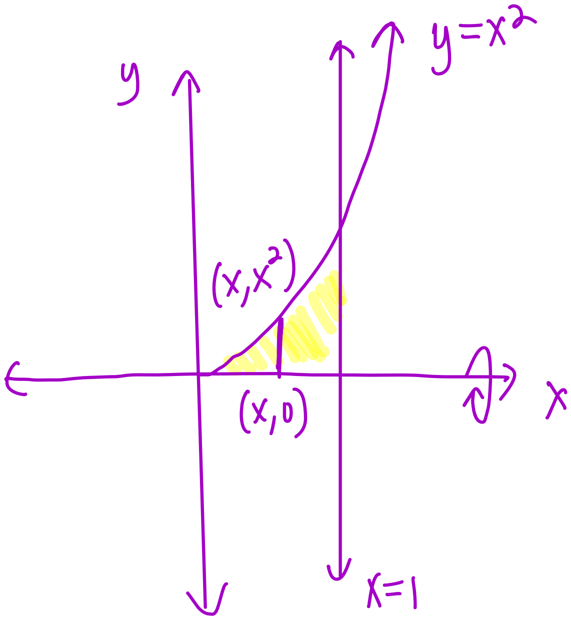
$$\lim_{n\to\infty} \sum_{k=1}^{n} \pi f(x_k) \Delta x$$

Volume of Solid:

$$\int_a^b \pi (f(x))^2 dx = \pi \int_a^b (f(x))^2 dx$$

When you integrate area you get volume, just like when we integrated height we got area.
The representative rectangle in the disk method is always perpendicular to the axis of revolution.

1. Find the volume generated when $y = x^2$ from $x = 0$ to $x = 1$ is revolved about the x-axis.

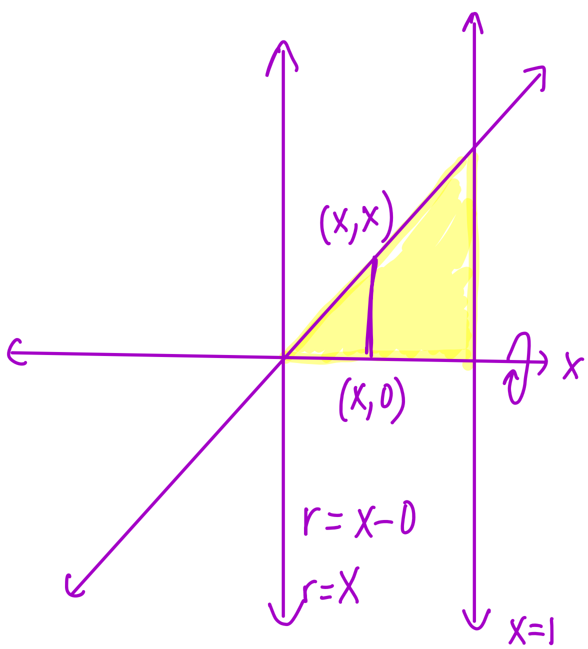


$r = x^2 - 0 = x^2$

$$V = \pi \int_0^1 (x^2)^2\,dx = \pi \int_0^1 x^4\,dx$$

$$= \pi \left[\frac{x^5}{5} \Big|_0^1 \right] = \frac{\pi}{5}$$

2. Find the volume of the solid that results from revolving the region bounded between $y = x$ and the x-axis from $x = 0$ to $x = 1$ around the x-axis.



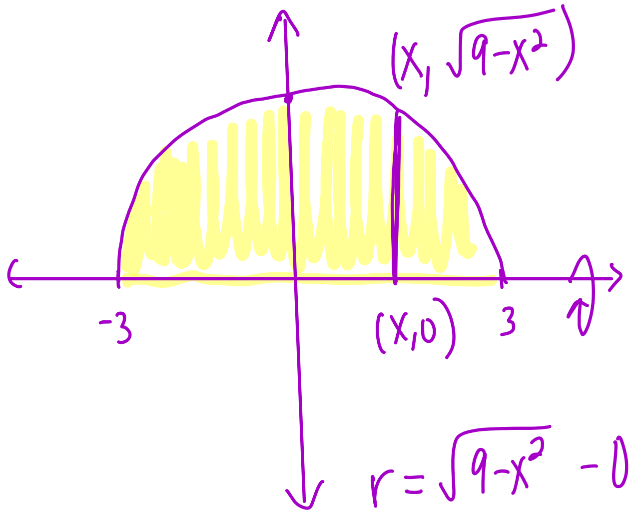
$$V = \pi \int_0^1 x^2\,dx$$

$$V = \pi \left[\frac{x^3}{3} \Big|_0^1 \right] = \frac{\pi}{3}$$

$x^2+y^2=9$

$y^2=9-x^2$

3. Find the volume of the solid that results when the region bounded by $y=\sqrt{9-x^2}$ and the x-axis is revolved around the the x-axis.



$r=\sqrt{9-x^2}-0$

$r=\sqrt{9-x^2}$

$V=\pi\int_{-3}^{3}\left(\sqrt{9-x^2}\right)^2dx$

$V=\pi\int_{-3}^{3}(9-x^2)\,dx$

$V=\pi\left[9x-\frac{x^3}{3}\Big|_{-3}^{3}\right]=36\pi$

DAY 2

Do Now:

1. Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=2$ and $x=0$ about the line $y=2$.

Horizontal Strip:

Classwork/Homework 03-19

Name: ______________________________

AP Calc – Definite Integrals, Limits of Riemann Sums, and SNACK STEALING?!

A snack stealing scandal has rocked RHS!

The math department is known for their love of food, but this time someone has gone too far! You have been asked to help solve the mystery of the missing snacks!

Answer the 6 multiple choice questions posted around the room and you will found out:

WHO: Carman

WHEN: Period 1

WHAT: Reese's PB Cups and Hot Chocolate

WITH: Compass

WHERE: Testing Center

WHO?

Which of the limits is equivalent to the following definite integral?

$$\int_0^{\pi} \cos x \, dx$$

$$\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$$

$$f(a + \Delta x k)$$

$$f(0 + \tfrac{\pi}{n} k) = \cos(\tfrac{\pi}{n} k)$$

Ⓐ $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{i}{n}\right) \cdot \frac{\pi}{n}$ STACK

~~Ⓑ~~ $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi i}{n}\right) \cdot \frac{i}{n}$ LEE

Ⓒ $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi i}{n}\right) \cdot \frac{\pi}{n}$ CARMAN

~~Ⓓ~~ $\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{i}{n}\right) \cdot \frac{i}{n}$ LOUGHRAN

WHEN?

Which of the limits is equivalent to the following definite integral?

$$\int_{-2}^{3} (x+1)\,dx$$

$\Delta x = \frac{3-(-2)}{n} = \frac{5}{n}$

$f(a + \Delta x k)$

$f(-2 + \frac{5}{n}k) = -2 + \frac{5}{n}k + 1 = -1 + \frac{5}{n}k$

- (A) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n} - 1\right) \cdot \frac{5}{n}$ PERIOD 1
- (B) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i-1}{n}\right) \cdot \frac{5}{n}$ PERIOD 3
- (C) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i+1}{n} + 1\right) \cdot \frac{5}{n}$ PERIOD 5
- (D) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i}{n} + 1\right) \cdot \frac{5}{n}$ PERIOD 7

WHAT? (PART 1)

Which of the definite integrals is equivalent to the following limit?

$a + \Delta x i$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \ln\left(2 + \frac{5i}{n}\right) \cdot \frac{5}{n}$$

Δx

$\frac{5}{n} = \frac{b-2}{n}$

$7 = b$

Ⓐ $\int_0^7 \ln x \, dx$ ICE CREAM

Ⓑ $\int_2^5 \ln x \, dx$ CUPCAKES

Ⓒ $\int_2^7 \ln x \, dx$ REESES PB CUPS

Ⓓ $\int_0^5 \ln x \, dx$ PEPPERMINT PATTIES

WHAT? (PART 2)

Which of the definite integrals is equivalent to the following limit?

$a + \Delta x i$ $\qquad \Delta x$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{\pi}{2} + \frac{\pi i}{2n}\right) \cdot \frac{\pi}{2n}$$

$a = \frac{\pi}{2}$

$\frac{\pi}{2n} = \frac{b - \frac{\pi}{2}}{n} \frac{(2)}{(2)}$

$\frac{\pi}{2n} = \frac{2(b - \frac{\pi}{2})}{2n}$

$\pi = 2b - \pi$

$2\pi = 2b$

$\pi = b$

Ⓐ $\int_0^{\pi} \cos x \, dx$ GREEN TEA

Ⓑ $\int_{\pi/2}^{3\pi/4} \cos x \, dx$ ICED CARAMEL LATTE

Ⓒ $\int_0^{\pi/2} \cos x \, dx$ PEACH SELTZER

Ⓓ $\int_{\pi/2}^{\pi} \cos x \, dx$ HOT CHOCOLATE

WITH?

constent function

$f(x) = 4$

Which of the definite integrals is equivalent to the following limit?

$$\lim_{n\to\infty} \sum_{i=1}^{n} 4 \cdot \frac{5}{n}$$

$\frac{5}{n} = \frac{b-0}{n}$

$5 = b-0$

$5 = b$

in all choices $a=0$

Ⓐ $\int_0^4 5x\,dx$ PAPER CLIP

Ⓑ $\int_0^5 4x\,dx$ SCREWDRIVER

Ⓒ $\int_0^4 5\,dx$ HAMMER

Ⓓ $\int_0^5 4\,dx$ COMPASS

WHERE?

$f(x) = \sqrt{x}$

$f(4 + \frac{5i}{n}) = \sqrt{4 + \frac{5i}{n}}$

Which of the definite integrals is equivalent to the following limit?

$\downarrow f(a + \Delta x i)$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{4 + \frac{5i}{n}} \cdot \frac{5}{n} \quad \Delta x$$

$a + \Delta x i$

$a = 4$

$\frac{5}{n} = \frac{b-4}{n}$ $\quad 5 = b - 4$

$b = 9$

Ⓐ $\int_0^4 \sqrt{4+x}\,dx$ LIBRARY

$f(x) = \sqrt{4+x}$

Ⓑ $\int_0^5 \sqrt{x}\,dx$ MAIN OFFICE

Ⓒ $\int_4^9 \sqrt{x}\,dx$ TESTING CENTER

Ⓓ $\int_4^9 \sqrt{4+x}\,dx$ SCIENCE STUDY CENTER

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

d) $N(t) = 20 + \int_0^t r(x)\,dx - .7t$

$N'(t) = r(t) - .7$

$A = 33.01329..$

$B = 166.579..$

t	$N(t)$
0	20
A	3.803
B	158.070
300	80

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\dfrac{t}{100}\right)^3\left(1 - \dfrac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

$t = 33.01329...$

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

(a) $\int_0^{300} r(t)\,dt = 270$

(b) $20 + \int_0^{300} (r(t)dt - .7)\,dt = 20 + \int_0^{300} r(t)\,dt - .7x\Big|_0^{300} = 80$

(c) There are 80 people in the line at time $t = 300$ (we found that in part b)

$300 + \frac{80}{.7} = 414.286$ or 414.285 seconds

2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

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a) $a(3) = -2.118$

b) $\int_0^3 v(t)dt = x(3) - x(0)$

$\int_0^3 v(t)dt = x(3) - (-5)$

$x(3) = \int_0^3 v(t)dt + 5 = 1.760$

c) $\int_0^{3.5} v(t)dt = 2.844\ (2.843)$ displacement over $[0, 3.5]$

$\int_0^{3.5} |v(t)|dt = 3.737$ total distance traveled by the particle $[0, 3.5]$

d) $v_2(t) = 2t - 1$

$2t - 1 = \dfrac{10\sin(.4t^2)}{t^2 - t + 3}$

$t = 1.571\ (1.570)$

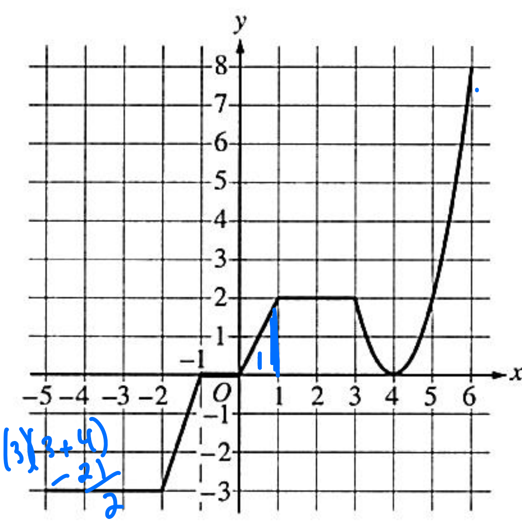
CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



$-\frac{1}{2}(3)(3+4)$

Graph of g

a) $\int_{-5}^{1} g(x)\,dx = f(1) - f(-5)$

$f(-5) = f(1) - \int_{-5}^{1} g(x)\,dx$

$= 3 - \left(\frac{-19}{2}\right)$

$g = f' \therefore f = \int g$

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$2(x^2 - 8x + 16)\,dx$

(b) Evaluate $\int_1^6 g(x)\,dx$. $= \int_1^3 g(x)\,dx + \int_3^6 2(x-4)^2\,dx = 4 + \left(2\left(\frac{x^3}{3} - 4x^2 + 16x\right)\right)\Big|_3^6 = 10$

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

f' + ↗ $(0,1) \cup (4,6)$

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

f'' + to − or − to + $x = 4$

f' ↗ ↘ or ↘ ↗

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$\frac{H(7)-H(5)}{7-5} = \frac{11-6}{2} = \frac{5}{2}$ m/yr rate of change in height of tree in m/yr at $t=6$ yrs

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

By MVT since the average rate of change of H on $(3,5)$ is 2 there must be at least one c $(3,5)$ where $H'(c) = 2$

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

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c) Average height of tree $= \frac{1}{10-2}\int_2^{10} H(t)\,dt = \frac{1}{8}\left[\frac{1}{2}(1)(1.5+2)+\frac{1}{2}(2)(2+6)+\frac{1}{2}(2)(6+11)+\frac{1}{2}(3)(11+15)\right]$ m

d) $G(x)=50,\ x=1$ bc →

$50 = \frac{100x}{1+x}$

$50+50x=100x$

$50=50x$

$1=x$

$\frac{dG(x)}{dt} = \frac{(1+x)(100\frac{dx}{dt}) - 100x(\frac{dx}{dt})}{(1+x)^2}$

$= \frac{2(100)(.03) - 100(.03)}{2^2}$

$\frac{6-3}{4} = \frac{3}{4}$ m/yr